MEMO

To: Old runners, people interested in old runners, race officials

From: Ray C. Fair

Subject: How Fast Do Old Men Slow Down

Date: May 28, 1991

The enclosed paper presents estimates of the rate at which men slow down with age in track, field, and road racing events. The paper is fairly technical, but Tables 2, 3, and 5 can be understood without reading the technical material. Table 2 shows that the age-factors in Masters Age-Graded Tables (MAGT) are excessively variable and biased against older runners. Tables 3 and 5 present the age-factors implied by this study.

INTeresting WORDING

Table 3 allows an old runner to estimate his projected times by age. To use the table, first pick the best time you think you could ever have run in the event, when you were in your 20's or early 30's. This could be either your actual time if you were running in these years or your best guess as to what this time would have been had you been running. Then multiply this time by the number in Table 3 that corresponds to your current age. This gives your projected time at your current age.

Race officials should consider using the age-factors in Table 3 in place of the age-factors in MAGT for age-graded races. The age-factors in Table 3 are more closely tied to actual best performances by age than are the MAGT age-factors, and they are not excessively variable.

Two examples of another way in which Table 3 can be used may be of interest. First, Mike Tymn in the February 1991 issue of National Masters News wrote about his 10K times. At the age of 41 he ran 31:38. He then had some bad years, but came back strong at the age of 53 to run 34:40. Given that he ran 31:38 at age 41, what would Table 3 predict he should run at age 53? The age-factor in Table 3 is 1.0791 for age 41 and 1.1889 for age 53, which is a 10.175 percent increase between the two ages. Tymn's age-53 time is thus predicted to be 10.175 greater than his age-41 time. This would be a time of 34:51, which is quite close to his actual time at age 53 of 34:40. Table 3 thus predicts very well in this case. To see how Tymn might do ten years hence, the age-factor for age 63 in Table 3 is 1.3135, which is a 10.48 percent increase from age 53. Given his age-53 time of 34:40, this would be a time at age 63 of 38:18.

The other example concerns Frank Shorter. Shorter has run the 4.55-mile Bemis-Forslund Pie Race many times, the first when he was 15. His best time occurred at age 32, when he ran 20:54. Last year at age 43 he ran 22:40. (See Runner's World, May 1991, p. 118, for these numbers.) What would Table 3 predict he should have run at age 43? The age-factor in Table 3 for age 43 is 1.0965. If we assume that Shorter's age-32 time of 20:54 is the best he could ever have run, his predicted time for age 43 is 1.0965 times 20:54, which is a time of 22:55. This is quite close to his actual time at age 43 of 22:40, and so again Table 3 predicts well. The age-factor for age 80 in Table 3 is 1.6568, which means that Shorter should be able to run the Pie Race when he is

80 in a time of 34:38 (1.6568 times 20:54) assuming that he is not sick or injured and that he stays in top shape for his age.

Table 5 presents the age-factors for the field events. These again may be of interest to race officials organizing age-graded meets.

Finally, a word of caution. The age-factors in Tables 3 and 5 are less reliable after about age 80 than they are before. The age-factors after age 80 will probably change the most in the future when the equations used in this study are reestimated in light of new age records that are set. Most likely the age-factors after age 80 will come down, but it is unclear at this time whether they will come down a lot or only a little.

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HOW FAST DO OLD MEN SLOW DOWN?

ABSTRACT

This study uses data on men's track and field and road racing records by age to estimate the rate at which men slow down with age. For most of the running events (400 meters through the half marathon), the slowdown rate per year is estimated to be .80 percent between ages 35 and 51. At age 51 the rate begins to increase. It is 1.04 percent at age 60, 1.46 percent at age 75, and 2.01 percent at age 95. The slowdown rate is smaller for 100 meters. For the events longer than the half marathon, the rate is smaller through about age 60 and then larger after that. The slowdown rate is generally larger at all ages for the field events.

Table 2 shows that the age-factors in Masters Age-Graded Tables are excessively variable and biased against older runners. Tables 3 and 5 present the age-factors implied by this study. These tables can be used to estimate one's projected time or distance by age. They can also be used by race officials for age-graded events. A brief comparison of the present results to results in the physiological literature is also presented in this paper.

The main estimation technique used is a combination of the polynomial-spline method and the frontier-function method. A number of the events have been pooled to provide more efficient estimates.

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HOW FAST DO OLD MEN SLOW DOWN?

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I. Introduction

This paper uses data on men's track and field and road racing records by age to estimate the rate at which men slow down with age. Eight track, eight field, and eleven road racing events are considered. The track events range from 100 meters to 10,000 meters, and the road racing events range from 5 kilometers to the marathon. The field events are the high jump, pole vault, long jump, triple jump, shot put (16 pounds), discus throw (2 kilograms), hammer throw (16 pounds), and javelin throw (800 grams).

Sections II - V consider the track and road racing events. Section II discusses the methodology that was followed, and Section III presents the estimation results. Section IV compares the age-factors published in Masters Age-Graded Tables (MAGT) with the age-factors implied by this study. It will be seen that the MAGT age-factors seem to be excessively variable and to be biased against older runners. Table 3 presents the age-factors implied by this study. This table should be of interest to old runners and to race officials organizing age-graded races. Section V provides a brief comparison

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[1]I am indebted to Don Andrews, William Brainard, Gregory Chow, Stephen Goldfeld, Ethan Nadel, and Christopher Sims for helpful comments and to Alvin Klevorick for pointing out an error in the data. I am also indebted to Peter Mundle for helpful discussions about the track data and to Basil and Linda Honikman at TACSTATS for supplying me with the road racing data and answering various questions about the data.

of the present results to results in the physiological literature. Section VI presents the results for the field events, and Table 5 presents the age-factors for the field events implied by this study.

II. The Methodology

The Model

For a given track or road racing event, let q denote the log of a runner's time in the race. For all runners of a given age, the theoretical frequency distribution for q probably looks something like that depicted in Figure 1. b, the lower bound, is the fastest time that could ever be run by a man of that age. Think of b as the biological limit of men, given perfect race conditions (but no tail winds allowed) and the use of the best training methods and equipment possible (but no performance enhancing drugs allowed). m is the median of the distribution, and u is the upper bound.[2]

This paper focuses on b. Let b_k denote the lower bound for runners of age k. One would expect b_k when plotted against k to look something like that depicted in Figure 2. (Remember that times are measured in logs, so the rates of change are percentage rates of change.) b_k is infinite for small babies, falls to some minimum at age k_1, stays at this minimum to age k_2, and then begins to rise. After b_k begins to rise, one would expect that the rate of slowing down is fairly constant through a certain age k_3 and then begins to rise. k_4 in the figure is the oldest age at which anyone could finish the race.

[2]If runners are included in the population who do not finish the race, then u might be considered to be infinite. This paper does not use u in the analysis, and so it does not matter for present purposes what is assumed about u.

FIGURE 1

Theoretical Frequency Distribution for q.

q = log of time

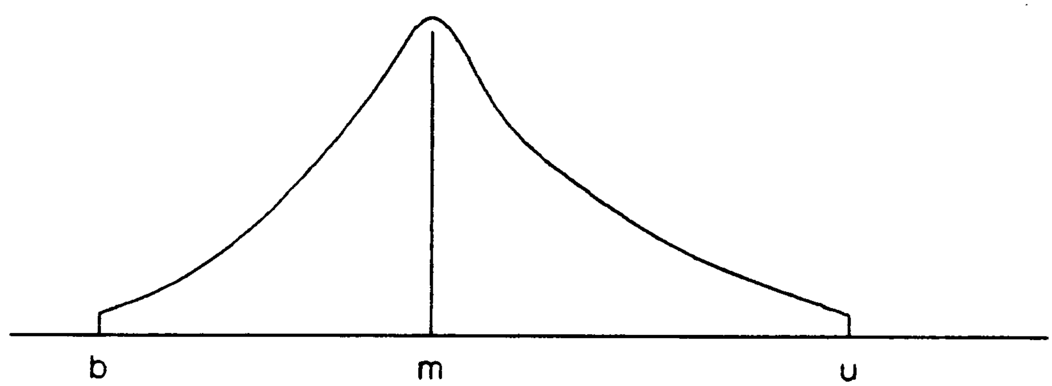
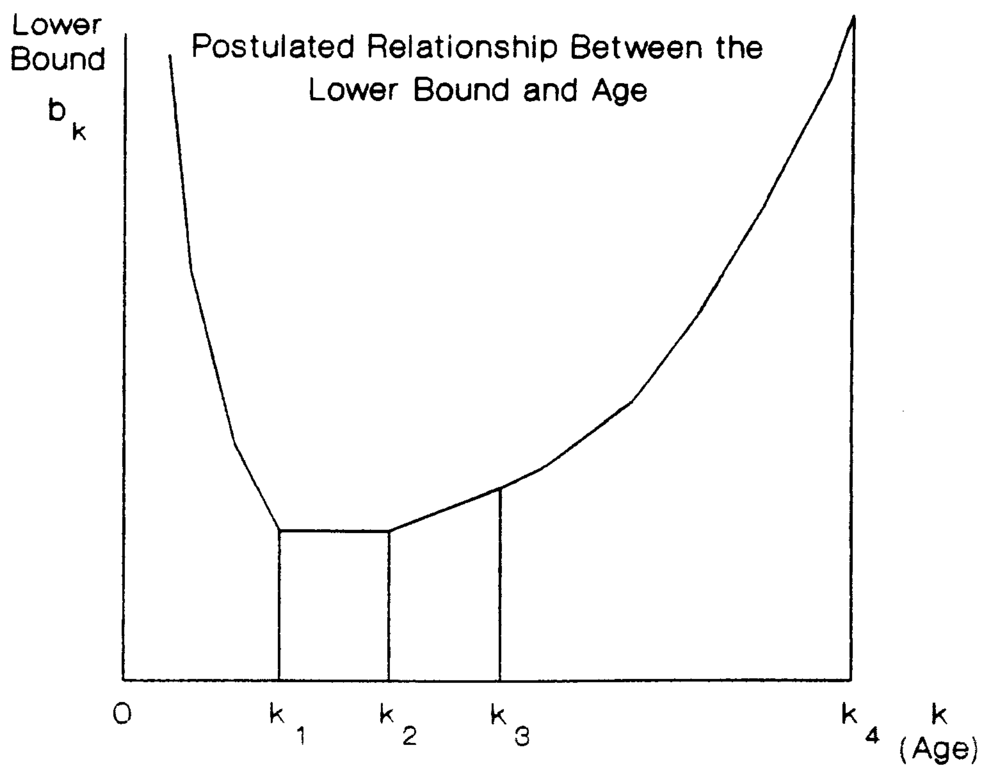


FIGURE 2



Postulated Relationship Between the Lower Bound and Age

The purpose of this paper is to estimate the function in Figure 2 from sometime after age k_2 on. The starting age used in the empirical work is 35, which means that k_2 is assumed to be less than or equal to 35. k_2 need not be equal to 35. If it is not, this just means that the sample used in this paper picks up the line sometime after k_2.

The functional form in Figure 2 is assumed in the empirical work to be linear between k_2 and k_3 and quadratic after that. At k_3, the linear and quadratic curves are assumed to touch and to have the same first derivative. The specification is:

$$(1) \quad b_k = \begin{cases} \alpha_1 + \alpha_2 k & \text{for } k_2 \le k \le k_3 \\ \alpha_3 + \alpha_4 k + \alpha_5 k^2 & \text{for } k > k_3 \end{cases},$$

with the restrictions

$$(2) \quad \begin{aligned} \alpha_3 &= \alpha_1 + \alpha_5 k_3^2 \\ \alpha_4 &= \alpha_2 - 2\alpha_5 k_3 \end{aligned} .$$

The two restrictions force the curves to touch and to have the same first derivative at k_3.[3] The unrestricted parameters to be estimated are α_1, α_2, α_5, and k_3.

The Data

The track data are from Masters Age Records For 1990, and the road racing data are from TACSTATS/USA. The track data give the current world record by age for each event. The road racing data give the current best time

[3]These restrictions are examples of polynomial spline restrictions. See Poirier (1976) for a general discussion of polynomial splines.

by an American for each event (data on world records by age are not yet available for road racing). Let r_k denote the log of the observed record time for age k for a given event, and let ϵ_k denote the difference between r_k and the unobserved b_k. r_k can thus be written:

$$(3) \quad r_k = b_k + \epsilon_k \ .$$

ϵ_k is the measurement error for r_k.

In principle ϵ_k can be either negative or positive, although negative measurement error does not seem likely. Two possible reasons for negative measurement error are 1) the true distance of the race is shorter than the stated distance and 2) the time is recorded wrong in favor of the runner. These kinds of errors are likely to be small because the races and records are monitored closely.

The story is different, however, regarding positive measurement error. The relevant question to consider is how many races for a given event have to be run by runners of age k before r_k becomes a good estimate of b_k? Let N_k denote the (unobserved) number of men age k who have run the particular event in question up to the current time. If N_k is in the millions, as it may be for runners in their 30's and 40's, there is probably a good chance that one has sampled close to the theoretical lower bound. If, on the other hand, N_k is only in the thousands or tens of thousands, as it probably is for very old runners, one is not likely to have sampled close to the lower bound. In fact, it is commonly stated that there are now many more runners, say, age 50 than there used to be, and as these runners age, the age records are likely to fall considerably. In 1989, nine age records in the 100 meters were broken, six of these for ages over 80. Eleven age records in the 10,000 meters were broken, seven of these for ages over 60. Results for other events are similar.[4] The large number of records broken in a single year indicates that the lower bound is far from being observed for many ages. This problem of not having a large enough sample at the higher ages to get a good estimate of the lower bound will be called the "small N_k" problem.[5]

Two adjustments were made in the data to try to account for the small N_k problem. First, the above theory postulates that after age k_2, b_k is greater than b_{k-i} for i positive (men slow down with age). Therefore, if r_k is greater than r_{k+i} for any positive i, r_k must have a relatively large positive measurement error associated with it. Observations of this kind, where the time for a given age is greater than the time for some older age, were not used.

Second, observations at very high ages were not used. The ages not used were always over 78 and in most cases over 81. The highest age used was 89, for 100 meters. The age cutoffs were chosen at the point where there was a large increase in the record time from one age to the next relative to the sizes of the previous increases. In discarding these observations it is implicitly assumed that the slow times are due to the small N_k problem and not to the fact that there is actually a large jump at that age. In other words, the problem is assumed to be a sampling problem, not a biological characteristic.

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[4]Compare the records in Masters Age Records 1990 with those in Masters Age Records 1989.

[5]The reason women were not considered in this study is that the small N_k problem seems very serious for them. Societies have not generally encouraged old women to run in track meets and road races.

These two adjustments may not be enough to completely eliminate the small N_k problem, and so the following results may be biased in the sense of overestimating the slowdown rate, especially at the older ages. An interesting question for future research is whether more can be done with the current data to try to adjust for the small N_k problem. It is the case, for example, that N_k is likely to be a decreasing function of k and that ϵ_k is a decreasing function of N_k. Therefore, ϵ_k is likely to be an increasing function of k. The approach taken in this study in dealing with this problem is simply to truncate the sample at the point where the size of the effect of k on ϵ_k appears to become large. An alternative approach would be to parameterize the function relating k to ϵ_k (say $\epsilon_k = \gamma_1 + \gamma_2 k + \gamma_3 k^2$ for k greater than some value $\bar{k}$), add this to (3), and try to estimate the new parameters (γ_1, γ_2, γ_3, $\bar{k}$) along with the others. My feeling is that the data are not good enough to allow anything sensible to come out of this, but it may be worth further thought.

Another possible approach is the following. Denote the density function in Figure 1 for a given age k as $f(q_k,\theta_k)$, where q_k is the log of the time in the event of an individual of age k and θ_k is a vector of parameters. Let q_k^{min} denote the minimum value of q_k in a sample of size N_k. q_k^{min} is an order statistic, and let $g(q_k^{min},\theta_k,N_k)$ denote its density function. The functional form of g depends, of course, on the functional form of f. The data used in this study are observations on q_k^{min} for k 35 and over. Given 1) observations on q_k^{min} , 2) an assumption about the functional form of f, 3) a parameterization of the elements of θ_k as functions of k, and 4) values for N_k or a parameterization of N_k as a function of k, one could estimate the parameters by maximum likelihood. Again, I doubt that the data are good enough to allow sensible estimates to be obtained using this approach, but it may be worth a try.

Until further work is done, the present results should be interpreted with caution. If the same estimation is done ten or twenty years hence, it is likely that the estimated slowdown rates will be smaller than the currently estimated rates. Whether they will be only slightly smaller or a lot smaller is the key open question.

Note finally that if all ages are getting better over time (say because of better nutrition, better training methods, or better equipment), this will not affect the estimated slowdown rates as long as all ages are getting better at the same rate. Progress like this will affect the estimated slowdown rates only if it differently affects the various ages.

The Econometrics

Let $d_k = 1$ if $k \leq k_3$ and $d_k = 0$ if $k > k_3$. Using this notation, substituting (1) into (3), and using the restrictions in (2) yields the equation to be estimated:

$$(4) \quad r_k = \alpha_1 + \alpha_2 k + \alpha_5(1-d_k)(k_3^2 - 2k_3 k + k^2) + \epsilon_k \quad , \ k = 35,\ldots K \ .$$

There are four parameters to estimate, α_1, α_2, α_5, and k_3, where it should be remembered that d_k is a function of k_3. K is the oldest age in the sample period. There are age gaps in the sample period because of the exclusion of observations with dominated times.

The main interest in this study is in the derivative of r_k with respect to k. This derivative is:

$$(5) \quad \partial r_k/\partial k = \alpha_2 + 2\alpha_5(1-d_k)(k - k_3) \quad .$$

This derivative is not a function of the constant term α_1 in (4), and so the size of the constant term is not of direct concern here.

Equation (4) pertains to a particular event. If one is willing to assume that α_2, α_5, and k_3 are the same across events, then the data on the different events can be pooled and more efficient estimates obtained. It does not seem unreasonable that the derivatives are the same at least for events close to each other in distance. When the data are pooled, different constant terms are needed for each event, since these obviously vary with distance. When the data were pooled for the results below, the following equation was estimated (n is the number of events pooled):

$$(6) \quad r_{ik} = \beta_1 D_{1ik} + \ldots + \beta_n D_{nik} + \alpha_2 k + \alpha_5(1-d_{ik})(k_3^2 - 2k_3 k + k^2) + \epsilon_{ik} \,, \qquad i = 1,\ldots,n \;\; ; \; k = 35,\ldots,K_i \,,$$

where r_{ik} is the log of the observed record for event i and age k, D_{jik} is a dummy variable that is equal to one when event i is equal to j and zero otherwise ($j = 1,\ldots,n$), $d_{ik} = 1$ if $k \le k_3$ and $d_{ik} = 0$ if $k > k_3$, ϵ_{ik} is the measurement error for event i and age k, and K_i is the oldest age used for event i. Again, there are age gaps in the sample period for a given event because of the exclusion of dominated observations. The n β_i coefficients in equation (6) pick up the n different constant terms.

Return now to the estimation of equation (4). Since positive measurement error for r_k is more likely than negative measurement error, the mean of ϵ_k is likely to be positive. If there is no negative measurement error at all, then $\epsilon_k \ge 0$ for all k. A positive mean for ϵ_k poses no problem in the estimation of equation (4) because the positive mean is merely absorbed in the estimate of the constant term. If the mean of ϵ_k is $\bar{\epsilon}$, define $\epsilon_k^* =$

$\epsilon_k - \bar{\epsilon}$, where ϵ_k^* has mean zero. Equation (4) can then be rewritten with ϵ_k^* replacing ϵ_k and the constant term changed from α_1 to $\alpha_1 + \bar{\epsilon}$. In this case α_1 is not identified, but this is of no concern here because the derivatives do not depend on α_1. One can thus estimate (4) by nonlinear least squares in the usual way. This estimation procedure will be called the NLS procedure.

There is, however, another estimation method that is of interest to consider. Under the assumption that $\epsilon_k \geq 0$ for all k, equation (4) can be estimated under the restriction that all estimated residuals are non negative. This procedure is common in the estimation of frontier production functions -- see, for example, Aigner and Chu (1968) and Schmidt (1976). The one added complication here is that equation (4) is nonlinear in coefficients. For linear equations the estimation problem can be set up as a quadratic programming problem and solved by standard methods, but for nonlinear equations some other procedure must be found.

The procedure used for the results below is the following. In the standard case the coefficients in equation (4) are estimated by minimizing the sum of squared residuals $\Sigma_{k=35}^{K}\hat{\epsilon}_k^2$. Instead, one can minimize a weighted sum $\Sigma_{k=35}^{K}\lambda_k\hat{\epsilon}_k^2$, where λ_k is equal to 1 if $\hat{\epsilon}_k \geq 0$ and is equal to a number greater than one if $\hat{\epsilon}_k < 0$. This penalizes negative errors more than non negative ones. For the work below a value of 100 was used for λ_k when $\hat{\epsilon}_k$ was less than zero. This was large enough to make nearly all the estimated residuals non negative at the optimum.[6] This estimation procedure will be called the "frontier" procedure.

[6]This procedure does not guarantee that all the estimated residuals are non negative because if $\hat{\epsilon}_k$ is negative but very close to zero, its contribution to the objective function is small even if λ_k is large. In practice the negative errors were very close to zero and were for all intents and purposes zero.

It turns out, as will be seen below, that the use of the frontier procedure over the NLS procedure has only a small effect on the estimated slope coefficients and thus on the estimated derivatives. The use of the frontier procedure primarily affects the estimate of the constant term, which is not of concern here.

An attempt was also made to estimate the parameters of (4) under the assumption that ϵ_k follows a gamma distribution, as discussed in Greene (1980). The use of this distribution has the advantage of allowing the statistical properties of the maximum likelihood estimator to be readily obtained, which the procedure discussed above does not. It also accommodates quite flexible shapes of the error distributions. Unfortunately, sensible results could not be obtained following this approach. The estimates of the two distribution parameters (P and λ in Greene's (1980) notation) were usually not sensible, and convergence was hard to obtain. It would be interesting to see in future work if this approach could be made to work, but the effort so far (which was considerable) was not successful.

III. The Results

The results of estimating the equation for each event by itself are presented first in Table 1. The estimates of α_2, k_3, and α_5 and their estimated standard errors are presented along with the implied values of the derivatives at ages 50, 60, 75, and 95. (The implied value of the derivative for ages below $\hat{k}_3$ is $\hat{\alpha}_2$.) The estimation technique for these results is NLS.

Set aside for the moment the 100 meter, 200 meter, and marathon events. Of the remaining 14 events, two stand out as being considerably different from the rest in Table 1: 10,000 meters and 5K. For 10,000 meters there is a small

TABLE 1

The Estimation Results for the Track and Road Racing Events

Event		$\hat{\alpha}_2$	SE($\hat{\alpha}_2$)	$\hat{k}_3$	SE($\hat{k}_3$)	$\hat{\alpha}_5$	SE($\hat{\alpha}_5$)	Derivative at age 50	60	75	95	SE	No. Obs.	Max Age
Track (meters)														
1	100	.0048	.0013	46.5	7.9	.00013	.00002	.0057	.0083	.0123	.0175	.015	29	89
2	200	.0076	.0003	65.8	1.8	.00056	.00012	.0076	.0076	.0179	.0403	.012	27	82
3	400	.0068	.0012	51.0	5.5	.00021	.00004	.0068	.0106	.0168	.0251	.016	25	81
4	800	.0052	.0083	37.1	37.2	.00012	.00002	.0083	.0107	.0143	.0192	.014	28	79
5	1500	.0088	.0006	54.0	4.4	.00018	.00004	.0088	.0109	.0162	.0233	.013	31	82
6	3000	.0080	.0008	52.8	4.1	.00024	.00005	.0080	.0114	.0186	.0281	.015	24	82
7	5000	.0087	.0009	50.1	6.3	.00013	.00003	.0087	.0112	.0150	.0201	.013	29	83
8	10000	.0089	.0014	47.3	17.8	.00006	.00004	.0092	.0105	.0125	.0150	.015	24	82
Road Racing														
9	5K	.0075	.0012	57.6	6.2	.00035	.00013	.0075	.0091	.0195	.0335	.027	24	82
10	10K	.0071	.0009	51.9	5.7	.00020	.00006	.0071	.0104	.0164	.0244	.020	31	81
11	15K	.0066	.0013	48.7	7.5	.00016	.00004	.0070	.0101	.0147	.0210	.016	28	82
12	10MI	.0066	.0031	45.1	19.6	.00011	.00005	.0077	.0100	.0134	.0180	.023	20	81
13	20K	.0052	.0044	42.2	21.9	.00014	.00005	.0073	.0100	.0141	.0196	.025	18	81
14	1/2MA	.0042	.0076	40.1	25.0	.00018	.00004	.0077	.0113	.0166	.0237	.029	22	81
15	30K	.0054	.0033	46.7	13.5	.00023	.00012	.0070	.0116	.0185	.0277	.032	12	78
16	20MI	.0055	.0024	49.1	9.4	.00028	.00011	.0059	.0115	.0198	.0309	.027	11	78
17	MA	.0063	.0009	58.2	2.5	.00061	.00012	.0063	.0085	.0269	.0515	.019	21	79
Pooled														
18[a]		.0069	.0006	47.7	3.0	.00016	.00001	.0076	.0109	.0157	.0221	.021	256	83
19[b]		.0057	.0018	49.0	7.5	.00026	.00009	.0062	.0115	.0194	.0299	.030	23	78
Frontier Method														
	Line[c]													
20	1	.0046	-	49.3	-	.00014	-	.0048	.0077	.0120	.0177	-	29	89
21	2	.0079	-	66.9	-	.00064	-	.0079	.0079	.0183	.0441	-	27	82
22	18	.0081	-	51.4	-	.00014	-	.0081	.0104	.0146	.0201	-	256	83
23	19	.0045	-	51.8	-	.00035	-	.0045	.0103	.0209	.0351	-	23	78
24	17	.0053	-	59.0	-	.00077	-	.0053	.0068	.0375	.0606	-	21	79

Notes:

[a]The pooled equations are 1-7,10-14.
[b]The pooled equations are 14,15.
[c]The frontier method used for the equation in this line above.

Max Age = Oldest age used in the sample period.

estimate of α_5, which means that the derivatives grow very slowly with age. For 5K the opposite is true. Note in particular that 10,000 meters is quite different from 10K even though it is the same distance, and likewise for 5K and 5,000 meters. I am inclined to discount the 10,000 meter and 5K results as likely reflecting considerable measurement error, given that they are so different from the rest.

The other two events that have somewhat different results are 30K and 20 miles. These both have slightly larger estimates of α_5 than the others. Two things could be going on here. First, it may be that at roughly the 30K distance, the slowdown rate at a given age begins to increase, and this is what the estimates are picking up. Second, the results may be unreliable. The 30K and 20 mile events are not as popular as the others, and so there is more of a potential small N_k problem here. The potential small N_k problem also reveals itself in the fact that the samples are small for these two events (12 and 11 observations respectively). The samples are small because many of the records were dominated by records at older ages and so were discarded. The high number of dominated records probably indicates a small N_k problem. It is thus an open question as to whether the 30K and 20 mile results are picking up an increase in the slowdown rate at a given age across distances or are simple due to a small sample problem.

The remaining five track events (400 meters through 5,000 meters) and five road racing events (10K through the half marathon) give similar results. There is no evidence of anything varying in a systematic way across distances. The implied derivatives at age 60 across the ten distances are in remarkably close agreement; the range is only .0100 at 10 miles and 20K to .0114 at 3000 meters. There is more variation in the estimates of α_2, where the range is

.0042 at the half marathon to .0087 at 5000 meters. The range at age 75 is .0134 at 10 miles to .0186 at 3000 meters, and the range at age 95 is .0180 at 10 miles to .0281 at 3000 meters. The estimated standard errors for $\hat{\alpha}_2$ and $\hat{k}_3$ are fairly large for some events.

Given that no systematic variation across distances is evident in the ten events, it seems sensible to pool them to obtain more efficient estimates. The results of doing this are reported in line 18 in Table 1. The estimate of α_2 is .0069, with an estimated standard error of .0006, and the estimate of k_3 is 47.7, with an estimated standard error of 3.0. The derivatives are .0076 at age 50, .0109 at age 60, .0157 at age 75, and .0221 at age 95.[7]

These pooling results are not sensitive to the exclusion of the 10,000 meters, 5K, 30K, and 20 mile events. When the observations from these events are included in the pooling, the estimates of α_2 and k_3 are .0069 and 48.3 respectively, and the derivatives at ages 50, 60, 75, and 95 are .0075, .0109, .0159, and .0227 respectively. These estimates are very close to the estimates presented in Table 1 when the four events are excluded.

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[7]Under the assumption that ϵ_k is normally distributed, which cannot be quite right because of the truncation issues, an F test can be used to test the hypothesis that α_2, α_5, and k_3 are the same across the ten events. There are 27 restrictions, and the number of observations in the pooled regression is 256. The F-value was 2.17, which compares with the critical value at the 1 percent level of 1.82, and so the hypothesis is rejected. Similar results were obtained when other sets of events were used. The hypothesis that the coefficients are the same across the specified events was usually rejected, although the computed F-values were usually not too much above the critical values. (The hypothesis that the coefficients are the same for 30K and 20 miles was, however, not rejected at the 5 percent level.)

I am not inclined to take these rejections as evidence against pooling. The computed F values were never too far from not rejecting the null hypothesis; the sample size is small relative to the number of restrictions; and there seems to be no compelling reason for believing that the coefficients change across the particular events, especially since no systematic patterns across the ten events were evident when the equations were estimated individually.

Consider now the other events. For 100 meters the results indicate that the rate of slowdown is smaller than it is for the other events. The estimated age at which the quadratic takes over is similar for 100 meters versus the pooled sample (46.5 versus 47.7), but the size of the derivatives are smaller. For example, at age 60 the slowdown rate is .0083 compared to .0109 for the pooled sample. At age 95 the rate is .0175 compared to .0221 for the pooled sample.

The results for 200 meters are quite different from the rest. The estimated age at which the quadratic takes over is 65.8, which is much larger than the other estimates. Also, the estimate of α_5 is much larger, which means that once the quadratic takes over, the estimated increase in the slowdown rate is larger than it is for the other events. The derivatives at age 60 are similar for 100 and 200 meters, but the derivative is noticeably larger at age 75 for 200 meters and considerably larger at age 95 (.0403 versus .0175). Because the 200 meter results stand out as being much different from the rest -- both from the 100 meter results and from the results for 400 meters and above -- they should be interpreted with considerable caution. It seems likely, for example, that the increase in the slowdown rate after age 64 has been overestimated.

Given that the results for 30K and 20 miles are similar to each other and differ somewhat from the rest, it is of interest to pool the two events. The results of this pooling are presented in line 19 in Table 1. Comparing lines 18 and 19, it can be seen that the estimated slowdown rate for 30K and 20 miles is less at the younger ages and more at the older ages. Although not shown in the table, the age at which the slowdown rate becomes greater for 30K and 20 miles is about 59. By age 95 the estimated slowdown rate is .0299 for

30K and 20 miles versus .0221 for the others.

The results for the marathon in line 17 continue the pattern of the estimated slowdown rate being less at the younger ages and more at the older ages. Although not shown in the table, the age at which the slowdown rate becomes greater for the marathon compared to the pooled events in line 18 is about 63. The estimated age at which the quadratic takes over is 58.2, which is higher than all the other estimates except the one for 200 meters. The estimate of α_2 is .0063, which means that until age 58 the estimated slowdown rate is constant at .63 percent per year. After age 58 the estimated slowdown rate picks up fairly rapidly (the estimate of α_5 is large), and by age 95 the derivative is by far the largest of any event at .0515. This derivative is even much larger than the derivative for the 30K and 20 mile events.

The differences between the marathon derivatives and the other derivatives at the older ages are large enough to make one question whether the marathon results should be trusted. There may be, however, more to the marathon than a mere 6.2 miles beyond 20 miles. Anyone who has run the last 6.2 miles in a marathon can appreciate this. If there is an important nonlinearity in going from 20 miles to the marathon, one might expect there to be a more rapid increase in the rate of slowing down at older ages for the marathon. This is what the current results show, although the estimated size of the effects should be taken with considerable caution.

The final estimates in Table 1 were obtained using the frontier procedure. Results are presented for 100 meters, 200 meters, pooled 400 meters through the half marathon, pooled 30K and 20 miles, and the marathon. The results using the frontier procedure are quite similar to the other results. None of the comparisons and conclusions discussed above are changed by the frontier results. Figure 3 shows a plot of the actual and predicted values for the marathon equation using the frontier procedure. The plots for the other events are similar in that the actual values are always close to or greater than the values on the frontier.

Finally, it should be mentioned that two other functional forms were tried. First, the quadratic in (1) was replaced with $b_k = \alpha_3 + \alpha_4/(k-\alpha_5)$ for $k > k_3$. The use of this form did not generally lead to as good fits as did the quadratic, and the curvature seemed too extreme at the top ages. Second, the quadratic was made more general by replacing the exponent 2 with a coefficient (α_6) to be estimated: $b_k = \alpha_3 + \alpha_4 k + \alpha_5 k^{\alpha_6}$. This allows the curvature to be either more or less extreme than that implied by the quadratic. This did not work because the estimates of α_5 and α_6 were too collinear for any confidence to be placed on the results. The estimates of α_6 were generally around 2, with large estimated standard errors.

IV. Age-Graded Tables

It is possible to use the coefficient estimates in Table 1 to estimate the ratio (denoted R_k) of the lower bound time at a given age k to the best lower bound time regardless of age. To do this, one needs a starting point, which in the present case is a value for R_{35}. Given R_{35}, R_{36} is $R_{35}(1+D_{36})$, where D_k is the derivative at age k computed from the estimated equation (remember that the derivatives are in percentage terms). R_{37} is then $R_{36}(1+D_{37})$, and so on.

The inverse of R_k is called an "age-factor" in Masters Age-Graded Tables (MAGT), and tables of age-factors are presented in MAGT for various events. Although MAGT does not explain how the age-factors were arrived at, it is of interest to see how they compare to the age-factors computed in this study.

Figure 3
Actual (*) and Predicted (+) Values for the Marathon

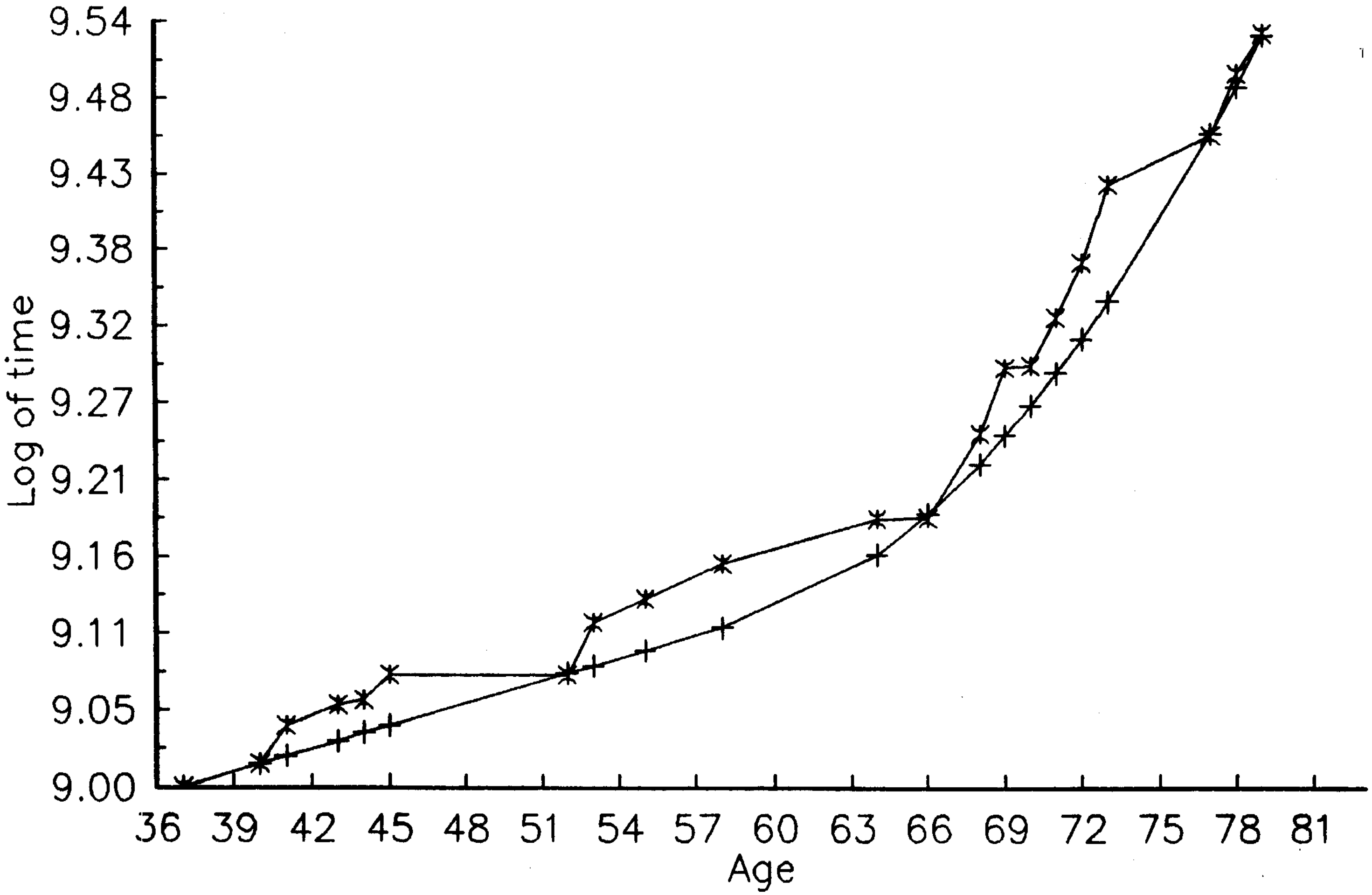

Table 2 presents the implied values of R_k (the inverse of the age-factors) from the table on page 24 in MAGT. These age-factors are for the 5K through half marathon events. The percentage changes in R_k are also presented in Table 2, along with the change in the percentage changes. These are the equivalent of the first and second derivatives of equation (4).

Table 2 shows that the MAGT value of R_k for age 35 is 1.02838. This means that MAGT has assumed that some loss in time has occurred by age 35 (2.838 percent to be exact).

Table 2 also presents values of R_k implied by the estimates in line 22 in Table 1. These are the estimates for the pooled events 400 meters through the half marathon, estimated by the frontier procedure. To make these values of R_k comparable to the MAGT values, the MAGT value of 1.02838 was used for R_{35} for the starting point. The derivatives from the equation and the changes in the derivatives are also presented in Table 2.

Two interesting conclusions emerge from in Table 2. First, the MAGT derivatives are (with one exception) increasing with age, but the sizes of the increases are erratic. The derivatives from this study, on the other hand, are constant through age 47 (actually 47.7) and then change at a constant amount (.00028) after that. This constant change is, of course, due to the use of the quadratic functional form. The erratic behavior of the change in the MAGT derivatives does not seem sensible. It seems unlikely, for example, that the derivative would change by .00033 at age 77, .00003 at age 78, .00050 at age 79, and .00004 at age 80. Nature is not generally like this.

The second conclusion is that the values of R_k are always higher for the present study. By age 90 the value of R_k is about 7 percent higher than the MAGT value. MAGT thus assumes that men slow down at a slower rate than seems

TABLE 2

Comparison of Estimated Age-Factors

		MAGT			Present Study			
	Age	R_k	D_k	$D_k - D_{k-1}$	R_k	D_k	$D_k - D_{k-1}$	
	35	1.02838	-	-	1.02838	-	-	
617	36	1.03455	0.00600	-	1.03666	0.00805	-	828
625	37	1.04080	0.00604	0.00004	1.04501	0.00805	0.00000	
643	38	1.04723	0.00618	0.00014	1.05343	0.00805	0.00000	
651	39	1.05374	0.00622	0.00004	1.06191	0.00805	0.00000	
671	40	1.06045	0.00636	0.00015	1.07047	0.00805	0.00000	
	41	1.06724	0.00640	0.00004	1.07909	0.00805	0.00000	
	42	1.07411	0.00644	0.00004	1.08778	0.00805	0.00000	
	43	1.08120	0.00660	0.00015	1.09654	0.00805	0.00000	
	44	1.08849	0.00675	0.00015	1.10537	0.00805	0.00000	
	45	1.09589	0.00679	0.00005	1.11428	0.00805	0.00000	
	46	1.10327	0.00673	-0.00006	1.12325	0.00805	0.00000	
	47	1.11086	0.00689	0.00016	1.13230	0.00805	0.00000	
	48	1.11882	0.00716	0.00027	1.14142	0.00805	0.00000	
	49	1.12714	0.00744	0.00028	1.15061	0.00805	0.00000	
	50	1.13585	0.00772	0.00028	1.15988	0.00805	0.00000	
	51	1.14482	0.00790	0.00018	1.16922	0.00805	0.00000	
	52	1.15420	0.00819	0.00030	1.17884	0.00823	0.00017	
	53	1.16401	0.00850	0.00030	1.18887	0.00850	0.00028	
	54	1.17412	0.00869	0.00019	1.19930	0.00878	0.00028	
	55	1.18469	0.00900	0.00032	1.21017	0.00906	0.00028	
	56	1.19589	0.00945	0.00044	1.22146	0.00933	0.00028	
	57	1.20744	0.00966	0.00021	1.23320	0.00961	0.00028	
	58	1.21936	0.00988	0.00022	1.24539	0.00989	0.00028	
	59	1.23153	0.00998	0.00010	1.25804	0.01016	0.00028	
	60	1.24409	0.01020	0.00023	1.27117	0.01044	0.00028	
	61	1.25691	0.01031	0.00011	1.28479	0.01071	0.00028	
	62	1.27000	0.01041	0.00011	1.29891	0.01099	0.00028	
	63	1.28370	0.01078	0.00037	1.31354	0.01127	0.00028	
	64	1.29769	0.01090	0.00012	1.32871	0.01154	0.00028	
	65	1.31199	0.01102	0.00012	1.34441	0.01182	0.00028	
	66	1.32679	0.01128	0.00026	1.36067	0.01209	0.00028	
	67	1.34210	0.01154	0.00026	1.37750	0.01237	0.00028	
	68	1.35777	0.01168	0.00013	1.39492	0.01265	0.00028	
	69	1.37382	0.01181	0.00014	1.41295	0.01292	0.00028	
	70	1.39043	0.01210	0.00028	1.43160	0.01320	0.00028	
	71	1.40726	0.01210	0.00001	1.45090	0.01348	0.00028	
	72	1.42470	0.01239	0.00029	1.47085	0.01375	0.00028	
	73	1.44259	0.01255	0.00016	1.49148	0.01403	0.00028	
	74	1.46113	0.01286	0.00031	1.51282	0.01430	0.00028	
	75	1.47995	0.01288	0.00002	1.53488	0.01458	0.00028	
	76	1.49925	0.01304	0.00017	1.55768	0.01486	0.00028	
	77	1.51930	0.01337	0.00033	1.58125	0.01513	0.00028	
	78	1.53965	0.01339	0.00003	1.60562	0.01541	0.00028	
	79	1.56104	0.01389	0.00050	1.63081	0.01569	0.00028	
	80	1.58278	0.01393	0.00004	1.65684	0.01596	0.00028	

TABLE 2 (continued)

	MAGT			Present Study		
Age	R_k	D_k	D_k-D_{k-1}	R_k	D_k	D_k-D_{k-1}
81	1.60514	0.01413	0.00020	1.68374	0.01624	0.00028
82	1.62840	0.01449	0.00037	1.71155	0.01651	0.00028
83	1.65235	0.01471	0.00021	1.74029	0.01679	0.00028
84	1.67701	0.01493	0.00022	1.76999	0.01707	0.00028
85	1.70242	0.01515	0.00023	1.80069	0.01734	0.00028
86	1.72861	0.01538	0.00023	1.83241	0.01762	0.00028
87	1.75593	0.01580	0.00042	1.86521	0.01790	0.00028
88	1.78380	0.01588	0.00007	1.89910	0.01817	0.00028
89	1.81291	0.01632	0.00044	1.93414	0.01845	0.00028
90	1.84264	0.01640	0.00008	1.97035	0.01872	0.00028

Notes:

MAGT - Masters Age-Graded Tables

R_k is the inverse of the age-factors in MAGT.

The MAGT age-factors are for the 5K through half marathon events.

The age-factors from the present study are from the estimates in line 22 in Table 1.

D_k is the percentage change in R_k: $D_k = (R_k/R_{k-1}) - 1$.

warranted by the data.

Table 3 presents five sets of values of R_k implied by the present study. The values are based on the coefficient estimates in lines 20-24 in Table 1, which were obtained using the frontier estimation procedure. The starting values of R_k (at age 35) are taken from MAGT. The values of R_k are presented through age 100, although the values for about age 83 and above are extrapolations beyond the end of the estimation period and should be interpreted with more caution.

As noted in Section III, the 200 meter results are somewhat suspect. If the 200 meter results are ignored, Table 3 shows that beginning with age 79 the values of R_k increase with the length of the race. At age 90 the values are 1.6979 for 100 meters, 1.9704 for 400 meters through the half marathon, 2.2169 for 30K and 20 miles, and 2.8573 for the marathon. If the best marathon time is taken to be 2 hours and 6 minutes, the value of R_k for the marathon implies that the best time for a 90 year old is 6 hours (2.8573 times 2 hours and 6 minutes). At age 100 the four values of R_k are, respectively, 2.0265, 2.4076, 3.1398, and 5.1821, although again these values are extrapolations way beyond the end of the estimation period.

Coming back to the MAGT values, although not shown in Table 2, the MAGT value of R_k at age 90 for 100 meters is 1.6736. This is again lower than the value of 1.6979 in Table 3, although in this case the values are quite close. The MAGT value of R_k at age 90 for the marathon is 1.8171, which is considerably lower than the value of 2.8573 in Table 3. The MAGT values imply that the slowdown rates for the marathon are smaller than they are for the 5K through half marathon events, which is opposite from what the empirical results seem to show and from what is presented in Table 3. Using a best marathon time of 2 hours and 6 minutes, the MAGT value of 1.8171 for age 90 implies that the best time for a 90 year old is 3 hours and 49 minutes, which compares to the above estimate of 6 hours using the results in this study.

Although the focus of this paper is on record times, Table 3 can be used by individual runners to estimate their age-adjusted times if one additional assumption is made. If in Figure 1 the difference between one's position on the horizontal axis and b does not change as b changes, then the results in Table 3 can be used. If this is true, it simply means that one's times are increasing at the same percentage rate as the record times are increasing. Obviously, injury or illness will increase one's distance from b. Also, if average runners slow down at a different rate from elite runners, then the distance from b for an average runner will be changing over time, thus making the results in Table 3 unreliable. Finally, if prolonged running wears parts of the body out -- the opposite situation from use-it-or-lose-it -- then one's distance from b will change over time as a function of how much past running has been done. This will also make the results in Table 3 unreliable.

Given the assumption that one's distance from b is constant over time and given an estimate of one's best time ever in the event, the values of R_k in Table 3 can be used to compute one's projected times by age. Race officials can also use the values to adjust each runner's time for his age.

V. Comparison to the VO_{2max} Results

A common measure of aerobic capacity in physiology is VO_{2max}. It is well known that VO_{2max} declines with age, and it is of interest to see how this decline compares to the decline in running performance estimated in this study. There seems to be nothing in the physiological literature for VO_{2max}

TABLE 3

Estimated Age Factors

R_k = Projected time for age k divided by overall best time
D_k = Percentage change in R_k

	100 meters		200 meters		400 meters - Half Marathon		30K, 20 miles		Marathon	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
35	1.0368	0.0046	1.0442	0.0068	1.0284	0.0080	1.0284	0.0045	1.0143	0.0053
36	1.0416	0.0046	1.0512	0.0068	1.0367	0.0080	1.0330	0.0045	1.0197	0.0053
37	1.0463	0.0046	1.0583	0.0068	1.0450	0.0080	1.0376	0.0045	1.0251	0.0053
38	1.0512	0.0046	1.0655	0.0068	1.0534	0.0080	1.0422	0.0045	1.0305	0.0053
39	1.0560	0.0046	1.0727	0.0068	1.0619	0.0080	1.0469	0.0045	1.0360	0.0053
40	1.0608	0.0046	1.0800	0.0068	1.0705	0.0080	1.0515	0.0045	1.0415	0.0053
41	1.0657	0.0046	1.0873	0.0068	1.0791	0.0080	1.0562	0.0045	1.0470	0.0053
42	1.0706	0.0046	1.0946	0.0068	1.0878	0.0080	1.0610	0.0045	1.0526	0.0053
43	1.0755	0.0046	1.1020	0.0068	1.0965	0.0080	1.0657	0.0045	1.0581	0.0053
44	1.0804	0.0046	1.1095	0.0068	1.1054	0.0080	1.0704	0.0045	1.0637	0.0053
45	1.0854	0.0046	1.1170	0.0068	1.1143	0.0080	1.0752	0.0045	1.0694	0.0053
46	1.0904	0.0046	1.1245	0.0068	1.1233	0.0080	1.0800	0.0045	1.0751	0.0053
47	1.0954	0.0046	1.1321	0.0068	1.1323	0.0080	1.0849	0.0045	1.0808	0.0053
48	1.1004	0.0046	1.1398	0.0068	1.1414	0.0080	1.0897	0.0045	1.0865	0.0053
49	1.1055	0.0046	1.1475	0.0068	1.1506	0.0080	1.0946	0.0045	1.0922	0.0053
50	1.1107	0.0048	1.1553	0.0068	1.1599	0.0080	1.0994	0.0045	1.0980	0.0053
51	1.1164	0.0051	1.1631	0.0068	1.1692	0.0080	1.1043	0.0045	1.1039	0.0053
52	1.1224	0.0054	1.1709	0.0068	1.1788	0.0082	1.1094	0.0046	1.1097	0.0053
53	1.1287	0.0056	1.1789	0.0068	1.1889	0.0085	1.1153	0.0053	1.1156	0.0053
54	1.1354	0.0059	1.1868	0.0068	1.1993	0.0088	1.1220	0.0060	1.1215	0.0053
55	1.1424	0.0062	1.1949	0.0068	1.2102	0.0091	1.1296	0.0067	1.1275	0.0053
56	1.1499	0.0065	1.2029	0.0068	1.2215	0.0093	1.1380	0.0074	1.1334	0.0053
57	1.1577	0.0068	1.2111	0.0068	1.2332	0.0096	1.1472	0.0081	1.1395	0.0053
58	1.1659	0.0071	1.2193	0.0068	1.2454	0.0099	1.1574	0.0089	1.1455	0.0053
59	1.1745	0.0074	1.2275	0.0068	1.2580	0.0102	1.1684	0.0096	1.1516	0.0053
60	1.1835	0.0077	1.2358	0.0068	1.2712	0.0104	1.1804	0.0103	1.1594	0.0068
61	1.1929	0.0079	1.2442	0.0068	1.2848	0.0107	1.1934	0.0110	1.1690	0.0083
62	1.2027	0.0082	1.2526	0.0068	1.2989	0.0110	1.2073	0.0117	1.1806	0.0099
63	1.2129	0.0085	1.2611	0.0068	1.3135	0.0113	1.2223	0.0124	1.1940	0.0114
64	1.2236	0.0088	1.2709	0.0078	1.3287	0.0115	1.2383	0.0131	1.2094	0.0129
65	1.2347	0.0091	1.2821	0.0088	1.3444	0.0118	1.2554	0.0138	1.2270	0.0145
66	1.2463	0.0094	1.2946	0.0098	1.3607	0.0121	1.2736	0.0145	1.2466	0.0160
67	1.2584	0.0097	1.3085	0.0108	1.3775	0.0124	1.2930	0.0152	1.2685	0.0175
68	1.2709	0.0100	1.3239	0.0118	1.3949	0.0126	1.3136	0.0159	1.2927	0.0191
69	1.2839	0.0102	1.3408	0.0128	1.4129	0.0129	1.3355	0.0166	1.3193	0.0206
70	1.2974	0.0105	1.3593	0.0138	1.4316	0.0132	1.3586	0.0174	1.3486	0.0222
71	1.3115	0.0108	1.3793	0.0147	1.4509	0.0135	1.3832	0.0181	1.3805	0.0237
72	1.3260	0.0111	1.4010	0.0157	1.4708	0.0138	1.4091	0.0188	1.4153	0.0252
73	1.3411	0.0114	1.4244	0.0167	1.4915	0.0140	1.4366	0.0195	1.4532	0.0268
74	1.3568	0.0117	1.4497	0.0177	1.5128	0.0143	1.4656	0.0202	1.4944	0.0283
75	1.3730	0.0120	1.4768	0.0187	1.5349	0.0146	1.4962	0.0209	1.5390	0.0298
76	1.3898	0.0123	1.5060	0.0197	1.5577	0.0149	1.5285	0.0216	1.5872	0.0314

TABLE 3 (continued)

	100 meters		200 meters		400 meters - Half Marathon		30K, 20 miles		Marathon	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
77	1.4072	0.0125	1.5372	0.0207	1.5813	0.0151	1.5626	0.0223	1.6395	0.0329
78	1.4253	0.0128	1.5705	0.0217	1.6056	0.0154	1.5986	0.0230	1.6960	0.0345
79	1.4440	0.0131	1.6062	0.0227	1.6308	0.0157	1.6365	0.0237	1.7570	0.0360
80	1.4633	0.0134	1.6442	0.0237	1.6568	0.0160	1.6765	0.0244	1.8229	0.0375
81	1.4834	0.0137	1.6848	0.0247	1.6837	0.0162	1.7187	0.0251	1.8941	0.0391
82	1.5041	0.0140	1.7281	0.0257	1.7115	0.0165	1.7631	0.0259	1.9710	0.0406
83	1.5255	0.0143	1.7742	0.0267	1.7403	0.0168	1.8099	0.0266	2.0541	0.0421
84	1.5477	0.0146	1.8233	0.0277	1.7700	0.0171	1.8593	0.0273	2.1438	0.0437
85	1.5707	0.0148	1.8756	0.0287	1.8007	0.0173	1.9113	0.0280	2.2407	0.0452
86	1.5944	0.0151	1.9312	0.0297	1.8324	0.0176	1.9662	0.0287	2.3455	0.0467
87	1.6190	0.0154	1.9904	0.0307	1.8652	0.0179	2.0240	0.0294	2.4587	0.0483
88	1.6444	0.0157	2.0534	0.0316	1.8991	0.0182	2.0849	0.0301	2.5812	0.0498
89	1.6707	0.0160	2.1204	0.0326	1.9341	0.0185	2.1491	0.0308	2.7138	0.0514
90	1.6979	0.0163	2.1917	0.0336	1.9704	0.0187	2.2169	0.0315	2.8573	0.0529
91	1.7260	0.0166	2.2677	0.0346	2.0078	0.0190	2.2883	0.0322	3.0129	0.0544
92	1.7551	0.0169	2.3484	0.0356	2.0465	0.0193	2.3637	0.0329	3.1815	0.0560
93	1.7852	0.0171	2.4345	0.0366	2.0865	0.0196	2.4432	0.0336	3.3645	0.0575
94	1.8162	0.0174	2.5260	0.0376	2.1279	0.0198	2.5272	0.0344	3.5631	0.0590
95	1.8484	0.0177	2.6235	0.0386	2.1707	0.0201	2.6158	0.0351	3.7790	0.0606
96	1.8817	0.0180	2.7275	0.0396	2.2149	0.0204	2.7094	0.0358	4.0137	0.0621
97	1.9161	0.0183	2.8382	0.0406	2.2607	0.0207	2.8082	0.0365	4.2692	0.0636
98	1.9516	0.0186	2.9562	0.0416	2.3080	0.0209	2.9127	0.0372	4.5475	0.0652
99	1.9884	0.0189	3.0821	0.0426	2.3569	0.0212	3.0231	0.0379	4.8510	0.0667
100	2.0265	0.0191	3.2165	0.0436	2.4076	0.0215	3.1398	0.0386	5.1821	0.0683

Notes:

The values for R_{35} are taken from MAGT.

The values for R_k are computed using the coefficient estimates in lines 20-24 in Table 1.

$D_k = R_k/R_{k-1} - 1$.

that is equivalent to Table 3, but there are some relevant results. Rogers et. al (1990) report a decline of 4.1 percent in 7.5 years in master athletes whose average age at the start was 62. This is a yearly fall of .0054, which compares to .0115 in Table 3 for age 64 and the events 400 meters - half marathon. Heath et. al. (1981) report between a 5 percent and 9 percent decline per decade for subjects between the ages of 50 and 62. A 5 percent decline is a yearly fall of .0049, and a 9 percent decline is a yearly fall of .0087. These numbers compare to .0096 in Table 3 for age 57. Both of these studies thus show a smaller VO_{2max} decline than the estimated decline in performance for the events 400 meters - half marathon in Table 3. Note in Table 3, however, that the derivative for the marathon is .0053 until age 60 and the derivative for 30K, 20 miles is .0045 until age 52. These numbers are close to the VO_{2max} results.

Dehn and Bruce (1972) provide an interesting regression to compare to the present results. Using a sample of ages between 40 and 69, they regress VO_{2max} adjusted for body weight on age. The coefficient estimate on age is -.362, and the estimate of the constant term is 52.741. One can compute from this regression the percentage fall in VO_{2max} at different ages, using the predicted value from this regression for the given age as the base value from which to compute the percentage. The results for selected ages compared to the results for 400 meters - half marathon in Table 3 are:

Age:	40	50	60	70	80	90	100
VO_{2max}:	.0095	.0105	.0117	.0132	.0152	.0180	.0210
Table 3:	.0080	.0080	.0104	.0132	.0160	.0187	.0215

The agreement in this case from age 60 on is remarkable, although for ages 40 and 50 the estimated decline in Table 3 is noticeably less than it is from the $\dot{V}O_{2max}$ regression. Also, estimates from the VO_{2max} regression for ages 50 and 60 are greater than the estimates from the two other studies reported above, and so the present comparisons are quite tentative. An interesting question for future work is whether the VO_{2max} results for the older ages (say 75 and above) can be used to help one estimate the slowdown rate at the older ages, where the small N_k problem is so severe.

VI. The Field Events

The small N_k problem is probably more serious for the field events than it is for the track and road racing events. This is particularly true for the shot put, discus throw, hammer throw, and javelin throw, where in many meets the weights of the relevant objects are less for older competitors. For this study only the results for the heaviest weights were used because these were the only results for which observations began at age 35.

The same procedure was followed for the field events as was followed for the other events. The log of the distance was used as the variable to be explained, and α_2 and α_5 are now expected to be negative since distance falls with age. Also, ϵ_k is expected to be mostly negative rather than mostly positive, and the frontier estimates are base on trying to force all the estimated residuals to be non positive rather than non negative. The estimation results are presented in Table 4.

The results for the high jump and triple jump are similar to each other. The are also similar to the results for the pooled sample in line 18 in Table 1, although the estimated slowdown rates are somewhat higher for the two field events. The estimated slowdown rates are considerably larger for the pole

TABLE 4

The Estimation Results for the Field Events

Event	$\hat{\alpha}_2$	SE($\hat{\alpha}_2$)	$\hat{k}_3$	SE($\hat{k}_3$)	$\hat{\alpha}_5$	SE($\hat{\alpha}_5$)	Derivative at age 50	60	75	95	SE	No. Obs.	Max Age
1 HJ	-.0093	.0009	51.5	6.2	-.00015	.00003	-.0093	-.0119	-.0163	-.0223	.017	26	90
2 PV	-.0130	.0010	64.1	2.1	-.00108	.00019	-.0130	-.0130	-.0366	-.0798	.036	31	86
3 LJ	-.0140	.0007	74.0	1.9	-.00158	.00030	-.0140	-.0140	-.0173	-.0805	.040	26	95
4 TJ	-.0125	.0012	53.1	9.6	-.00015	.00006	-.0125	-.0145	-.0189	-.0247	.022	27	83
5 SP	-.0281	.0010	-	-	-	-	-.0281	-.0281	-.0281	-	.061	23	80
6 DT	-.0280	.0013	-	-	-	-	-.0280	-.0280	-.0280	-	.070	20	78
7 HT	-.0275	.0009	-	-	-	-	-.0275	-.0275	-.0275	-	.049	21	76
8 JT	-.0273	.0010	-	-	-	-	-.0273	-.0273	-.0273	-	.059	26	77
Pooled													
9[a]	-.0278	.0005	-	-	-	-	-.0278	-.0278	-.0278	-	.060	90	80
Frontier Method													
Line[b]													
10 1	-.0095	-	62.7	-	-.00030	-	-.0095	-.0095	-.0170	-.0290	-	26	90
11 2	-.0129	-	65.8	-	-.00125	-	-.0129	-.0129	-.0358	-.0856	-	31	86
12 3	-.0135	-	75.4	-	-.00194	-	-.0135	-.0135	-.0135	-.0895	-	26	95
13 4	-.0129	-	60.5	-	-.00018	-	-.0129	-.0129	-.0180	-.0251	-	27	83
14 9	-.0266	-	-	-	-	-	-.0266	-.0266	-.0266	-	-	90	80

Notes:

[a]The pooled equations are 5-8.
[b]The frontier method used for this line above.

Max Age = oldest age used in the sample period.

HJ = high jump
PV = pole vault
LJ = long jump
TJ = triple jump
SP = shot put, 16 pounds
DT = discus throw, 2 kgs
HT = hammer throw, 16 pounds
JT = javelin throw, 800 grams

vault and the long jump, especially after the quadratic takes over at ages 64.1 and 74.0, respectively.

Sensible results using the quadratic specification could not be obtained for the other four field events -- the throwing events. The relationship between r_k and k appeared to be linear or close to linear up to about age 80, and there were not enough observations past age 80 to estimate the quadratic part with even moderate precision. There is, however, a remarkable similarity in results across the four throwing events when the linear specification is used. These results are presented in lines 5-8 in Table 4. The estimates of α_2 range only from -.0273 to -.0281. When the four events are pooled (line 9), the estimate of α_2 is -.0278. This estimated slowdown rate is larger than the rates for the other four field events expect for the pole vault and the long jump at the older ages. This estimated rate for the four throwing events seems relevant up to about age 80, but it should not be extrapolated beyond this. The data so far tell us little about what happens beyond age 80.

The frontier estimates for the first four field events and for the four throwing events pooled are presented in lines 10-14 in Table 4. As was the case for the track and road racing events, the differences between the NLS estimates and the frontier estimates are small, especially regarding the implied derivative values. The largest difference is for the high jump, where the estimate of k_3 is increased from 51.5 to 62.7 and the estimate of α_5 is changed from -.00015 to -.00030. Even here, however, the effects on the derivatives are fairly small.

The implied values of R_k for the first four field events and for the four throwing events pooled are presented in Table 5. Only values through age 80 are presented for the four throwing events pooled, for reasons discussed

TABLE 5

Estimated Age Factors for the Field Events

R_k - Projected time for age k divided by overall best time
D_k - Percentage change in R_k

	High Jump		Pole Vault		Long Jump		Triple Jump		Throwing Events	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
35	0.9381	-0.0095	0.9302	-0.0129	0.9328	-0.0135	0.9311	-0.0129	0.9381	-0.0266
36	0.9291	-0.0095	0.9182	-0.0129	0.9202	-0.0135	0.9191	-0.0129	0.9132	-0.0266
37	0.9203	-0.0095	0.9063	-0.0129	0.9078	-0.0135	0.9072	-0.0129	0.8889	-0.0266
38	0.9115	-0.0095	0.8946	-0.0129	0.8955	-0.0135	0.8955	-0.0129	0.8652	-0.0266
39	0.9028	-0.0095	0.8830	-0.0129	0.8834	-0.0135	0.8840	-0.0129	0.8423	-0.0266
40	0.8942	-0.0095	0.8716	-0.0129	0.8715	-0.0135	0.8725	-0.0129	0.8199	-0.0266
41	0.8857	-0.0095	0.8603	-0.0129	0.8597	-0.0135	0.8613	-0.0129	0.7981	-0.0266
42	0.8772	-0.0095	0.8492	-0.0129	0.8481	-0.0135	0.8502	-0.0129	0.7769	-0.0266
43	0.8689	-0.0095	0.8382	-0.0129	0.8366	-0.0135	0.8392	-0.0129	0.7562	-0.0266
44	0.8606	-0.0095	0.8274	-0.0129	0.8253	-0.0135	0.8284	-0.0129	0.7361	-0.0266
45	0.8524	-0.0095	0.8167	-0.0129	0.8142	-0.0135	0.8177	-0.0129	0.7166	-0.0266
46	0.8442	-0.0095	0.8061	-0.0129	0.8032	-0.0135	0.8071	-0.0129	0.6975	-0.0266
47	0.8362	-0.0095	0.7957	-0.0129	0.7923	-0.0135	0.7967	-0.0129	0.6790	-0.0266
48	0.8282	-0.0095	0.7854	-0.0129	0.7816	-0.0135	0.7864	-0.0129	0.6609	-0.0266
49	0.8203	-0.0095	0.7753	-0.0129	0.7710	-0.0135	0.7762	-0.0129	0.6433	-0.0266
50	0.8125	-0.0095	0.7652	-0.0129	0.7606	-0.0135	0.7662	-0.0129	0.6262	-0.0266
51	0.8047	-0.0095	0.7553	-0.0129	0.7503	-0.0135	0.7563	-0.0129	0.6096	-0.0266
52	0.7971	-0.0095	0.7456	-0.0129	0.7402	-0.0135	0.7466	-0.0129	0.5934	-0.0266
53	0.7894	-0.0095	0.7359	-0.0129	0.7302	-0.0135	0.7369	-0.0129	0.5776	-0.0266
54	0.7819	-0.0095	0.7264	-0.0129	0.7203	-0.0135	0.7274	-0.0129	0.5623	-0.0266
55	0.7745	-0.0095	0.7170	-0.0129	0.7106	-0.0135	0.7180	-0.0129	0.5473	-0.0266
56	0.7671	-0.0095	0.7077	-0.0129	0.7010	-0.0135	0.7088	-0.0129	0.5328	-0.0266
57	0.7598	-0.0095	0.6986	-0.0129	0.6915	-0.0135	0.6996	-0.0129	0.5186	-0.0266
58	0.7525	-0.0095	0.6896	-0.0129	0.6822	-0.0135	0.6906	-0.0129	0.5048	-0.0266
59	0.7453	-0.0095	0.6806	-0.0129	0.6729	-0.0135	0.6817	-0.0129	0.4914	-0.0266
60	0.7382	-0.0095	0.6718	-0.0129	0.6639	-0.0135	0.6729	-0.0129	0.4783	-0.0266
61	0.7312	-0.0095	0.6632	-0.0129	0.6549	-0.0135	0.6641	-0.0131	0.4656	-0.0266
62	0.7242	-0.0095	0.6546	-0.0129	0.6460	-0.0135	0.6552	-0.0134	0.4533	-0.0266
63	0.7172	-0.0097	0.6461	-0.0129	0.6373	-0.0135	0.6461	-0.0138	0.4412	-0.0266
64	0.7098	-0.0103	0.6378	-0.0129	0.6287	-0.0135	0.6370	-0.0141	0.4295	-0.0266
65	0.7020	-0.0109	0.6295	-0.0129	0.6202	-0.0135	0.6277	-0.0145	0.4181	-0.0266
66	0.6940	-0.0115	0.6210	-0.0134	0.6118	-0.0135	0.6184	-0.0149	0.4070	-0.0266
67	0.6855	-0.0121	0.6112	-0.0159	0.6035	-0.0135	0.6090	-0.0152	0.3961	-0.0266
68	0.6768	-0.0127	0.5999	-0.0184	0.5954	-0.0135	0.5995	-0.0156	0.3856	-0.0266
69	0.6678	-0.0133	0.5874	-0.0209	0.5873	-0.0135	0.5900	-0.0159	0.3754	-0.0266
70	0.6585	-0.0139	0.5736	-0.0234	0.5794	-0.0135	0.5804	-0.0163	0.3654	-0.0266
71	0.6489	-0.0145	0.5588	-0.0259	0.5716	-0.0135	0.5707	-0.0166	0.3557	-0.0266
72	0.6391	-0.0151	0.5430	-0.0284	0.5638	-0.0135	0.5611	-0.0170	0.3462	-0.0266
73	0.6290	-0.0158	0.5262	-0.0309	0.5562	-0.0135	0.5513	-0.0173	0.3370	-0.0266
74	0.6188	-0.0163	0.5087	-0.0333	0.5487	-0.0135	0.5416	-0.0177	0.3281	-0.0266
75	0.6083	-0.0170	0.4904	-0.0358	0.5413	-0.0135	0.5318	-0.0180	0.3193	-0.0266
76	0.5976	-0.0176	0.4716	-0.0383	0.5327	-0.0159	0.5220	-0.0184	0.3109	-0.0266

TABLE 5 (continued)

	High Jump		Pole Vault		Long Jump		Triple Jump		Throwing Events	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
77	0.5867	-0.0182	0.4524	-0.0408	0.5221	-0.0198	0.5122	-0.0187	0.3026	-0.0266
78	0.5757	-0.0188	0.4328	-0.0433	0.5098	-0.0237	0.5025	-0.0191	0.2945	-0.0266
79	0.5646	-0.0194	0.4130	-0.0458	0.4957	-0.0275	0.4927	-0.0194	0.2867	-0.0266
80	0.5533	-0.0200	0.3930	-0.0483	0.4802	-0.0314	0.4829	-0.0198	0.2791	-0.0266
81	0.5419	-0.0206	0.3731	-0.0508	0.4632	-0.0353	0.4732	-0.0202	-	-
82	0.5304	-0.0212	0.3532	-0.0533	0.4451	-0.0392	0.4635	-0.0205	-	-
83	0.5189	-0.0218	0.3335	-0.0558	0.4259	-0.0430	0.4538	-0.0209	-	-
84	0.5072	-0.0224	0.3141	-0.0582	0.4059	-0.0469	0.4442	-0.0212	-	-
85	0.4956	-0.0230	0.2950	-0.0607	0.3853	-0.0508	0.4346	-0.0216	-	-
86	0.4839	-0.0236	0.2764	-0.0632	0.3643	-0.0547	0.4251	-0.0219	-	-
87	0.4722	-0.0242	0.2582	-0.0657	0.3429	-0.0585	0.4156	-0.0223	-	-
88	0.4605	-0.0248	0.2406	-0.0682	0.3215	-0.0624	0.4062	-0.0226	-	-
89	0.4487	-0.0254	0.2236	-0.0707	0.3002	-0.0663	0.3968	-0.0230	-	-
90	0.4371	-0.0260	0.2072	-0.0732	0.2792	-0.0702	0.3876	-0.0233	-	-
91	0.4254	-0.0266	0.1915	-0.0757	0.2585	-0.0740	0.3784	-0.0237	-	-
92	0.4138	-0.0272	0.1766	-0.0781	0.2383	-0.0779	0.3693	-0.0240	-	-
93	0.4023	-0.0278	0.1623	-0.0806	0.2188	-0.0818	0.3603	-0.0244	-	-
94	0.3909	-0.0284	0.1488	-0.0831	0.2001	-0.0857	0.3513	-0.0248	-	-
95	0.3795	-0.0290	0.1361	-0.0856	0.1822	-0.0895	0.3425	-0.0251	-	-
96	0.3683	-0.0296	0.1241	-0.0881	0.1652	-0.0934	0.3338	-0.0255	-	-
97	0.3572	-0.0302	0.1129	-0.0906	0.1491	-0.0973	0.3252	-0.0258	-	-
98	0.3461	-0.0308	0.1024	-0.0931	0.1340	-0.1012	0.3167	-0.0262	-	-
99	0.3352	-0.0315	0.0926	-0.0956	0.1199	-0.1050	0.3083	-0.0265	-	-
100	0.3245	-0.0321	0.0835	-0.0981	0.1069	-0.1089	0.3000	-0.0269	-	-

Notes:

The values for R_{35} are taken from MAGT.

The values for R_k are computed using the coefficient estimates in lines 1-9 in Table 1.

$D_k = R_k/R_{k-1} - 1$.

above. The estimates in lines 10-14 in Table 4 were used for these values, which are the estimates based on the frontier procedure. The values for R_{35} for each event were taken from MAGT.

Comparing Tables 3 and 5, almost all the derivatives are larger in absolute value in Table 5. Men seem to slow down faster in the field events than they do in the track and road racing events. The two exceptions to this are 1) the high jump and triple jump at the older ages, where the slowdown rates are not out of line with the rates for the pooled events in Table 3, and 2) the marathon, where the slowdown rates at the older ages are high relative to the rates for the high jump, triple jump, and the throwing events. These two exceptions pertain only to ages beyond about 80, however, and it seems clear that for ages below 80 the slowdown rate is greater for the field events than it is for the running events.

V. Conclusion

Since I am an economist, one might ask if the above has anything to do with economics? From an economist's perspective, is this study simply an exercise in applying some econometric techniques to a data set? Maybe. But I am struck from looking at the numbers in Table 3 how small the slowdown rates are. For example, using the values of R_k for the events 400 meters - half marathon, a man of 85 is only 49 percent slower than he was at age 55 (1.8007 versus 1.2102). (Presumably the numbers are similar for women.) Do these numbers say something about productivity loss with age, about the optimal wage profile with age, about retirement policies? Maybe most societies are too pessimistic about the losses from aging and have passed various laws dealing with old people under incorrect assumptions? Perhaps old people can be used in more rigorous ways than has heretofore been the case? Or maybe the numbers in Table 3 are only of interest to old runners as they run ever more slowly into the sunset.

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Dear Al:

Thanks for your letter. I am not going to Turku, and so I thought I would pass along a few comments in this letter that may be of help to your deliberations.

1. The advantage of my numbers (Tables 3 and 5) is that they are based on actual best (world record) performances by age. In this sense they are much closer to actual experiences than are numbers based on people's guesses as to what someone of a given age may do some day. Also, the numbers do not rely on any one individual, who may have good and bad years, but on the best ever done by anyone of that age.

2. A statistical study like mine is subject to some uncertainty, and the numbers should not be taken as the absolute truth. (The paper discusses these uncertainties -- some technical, but some merely common sense.) Also, some of the numbers are likely to be more reliable than others, and I will now indicate which numbers I think you can trust and which are on shakier grounds.

a) Almost all the numbers in Tables 3 and 5 beyond age 80 are extrapolations beyond the ages used for the estimation of the equations, and so these numbers should be interpreted with more caution. The numbers though age 85 or 86 are probably fairly good, but beyond this things get more uncertain.

b) The most reliable results are those in Table 3 for 400 meters - Half Marathon, which are based on a pooled (larger) sample.

c) As noted in the paper, the 200 meter results are not very trustworthy after about age 65. The rate of slowdown after age 65 has probably been overestimated for the 200 meters.

d) The marathon results after about age 75 are fairly uncertain.

e) In almost all cases where there is some doubt about the numbers, the estimated slowdown rates are likely to be too large. In other words, if you adjust the numbers, you would want to make the slowdown rates smaller than the estimated rates in Tables 3 and 4.

3. I would be careful if I were you in making big adjustments to the numbers in Tables 3 and 5, especially below age 80. Any big adjustments that you make are pushing the numbers far away from actual experience, which does not seem sensible to me given the use to which the tables are put. If, say, I do this study over again 10 or 15 years from now, when many age records will have been broken, I will get different numbers for Tables 3 and 5. What I am saying is that the changes below age 80 are likely to be small (in this sense, the present numbers are reliable), and so you should not make big changes from the present numbers.

4. Note in the tables that D_k is the rate of slowdown at age k. The <u>change</u> in D_k, namely $D_k - D_{k-1}$, is an estimate of how much the slowdown rate changes from one age to the next. Consider Table 2. My estimates are that the slowdown rate is the same each year (namely .00805) until about age 51 (the actual estimated age is 51.4). From age 52 on, the slowdown rate increases by .00028 per year. (People get slower faster after about age 51 according to the estimates.) In my estimation work I have imposed the requirement that once the slowdown rate begins to increase, the rate of increase is constant each year. (This is called a quadratic specification.) This is why the .00028 number is the same each year from 52 on. Whatever numbers you end up choosing, I think you should impose this requirement. It does not make sense to have the MAGT D_k numbers changing in the erratic way that they do in Table 2.

5. Finally, you asked about the 400 meter results. My estimation results (Table 1) don't show 400 meters to be any different from 800 meters through the half marathon, and so I just pooled the 400 meters observations with the observations for these other events. The 100 meter and 200 meter results are different from the rest (and the 200 meter results are somewhat unreliable after age 65), but the 400 meter results did not seem different from 800 meters through the half marathon.

I hope these comments are of some help. If I can be of further help, either before or after your meeting, don't hesitate to call on me. The age-graded tables are a great idea, and I would like to see them as realistic as possible.

You asked how I got into this. I am a professor of economics at Yale. Most of my time is spent doing research of my own choosing. One of my subfields within economics is statistical methods (usually called econometric methods in economics), and these methods are the obvious ones to apply to the age-record data. Also, economists are becoming more and more interested in aging (as are people in other professions), and I am a runner. All this led to the current paper.

Sincerely,

Ray

Ray C. Fair
Professor of Economics

NATIONAL MASTERS NEWS

The official world and U.S. publication for Masters track & field, long distance running and race walking.

June 20, 1991

Ray Fair
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Box 2125, Yale Station
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Re: How Fast Do Old Men Slow Down?

Dear Ray:

Thank you for sending me a copy of your excellent paper on "How Fast Do Old Men Slow Down."

While I couldn't follow your equations, I understand Tables 3 and 5 and how it could apply to the question of how fast do we slow down.

I agree with you that the current Masters Age-Graded Tables are biased against older runners. When we developed the tables in 1989, we compromised between those, such as Pete Mundle, who felt the older-age factors should be tougher; and Chuck Phillips, who felt the older age factors should be easier. After two years of working with the factors in actual competitions, I concur with you and Phillips that the older-age factors should be eased.

Frankly, I was amazed then and continue to be amazed at how different inviduals, working independently from the same data, can arrive at such different conclusions.

A couple of questions: what prompted you to do this work? Is it voluntary? Or part of a project with your work? In table 3, your 200 factors are easier than your 400 factors. This is contrary to all previous assumptions that the 400 is the race where age takes its biggest toll.

In otherwords, your factor for M40 100, for example, is 1.0608; then you rise to 1.0800 for the 200; then drop back for the 400 and above. MAGT rises through the 400 before dropping back.

Do you really feel the factors should be the same for the 400 as for the half-marathon? Our experience tells us the rate of decline is less, the greater the distance. In Table 2, why does your D_k-D_{k-1} column show .0000 up to age 51, then jump to 17, then jump to .00028 and hold? Does that assume a dramatic dropoff at age 51? Why?

Enclosed is a copy of a letter I've sent to our Committee. Are you planning on being in Turku? Or Naperville?

Sincerely, *[signature]*

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HOW FAST DO OLD MEN SLOW DOWN?

ABSTRACT

This study uses data on men's track and field and road racing records by age to estimate the rate at which men slow down with age. For most of the running events (400 meters through the half marathon), the slowdown rate per year is estimated to be .80 percent between ages 35 and 51. At age 51 the rate begins to increase. It is 1.04 percent at age 60, 1.46 percent at age 75, and 2.01 percent at age 95. The slowdown rate is smaller for 100 meters. For the events longer than the half marathon, the rate is smaller through about age 60 and then larger after that. The slowdown rate is generally larger at all ages for the field events.

Table 2 shows that the age-factors in Masters Age-Graded Tables are excessively variable and biased against older runners. Tables 3 and 5 present the age-factors implied by this study. These tables can be used to estimate one's projected time or distance by age. They can also be used by race officials for age-graded events. A brief comparison of the present results to results in the physiological literature is also presented in this paper.

The main estimation technique used is a combination of the polynomial-spline method and the frontier-function method. A number of the events have been pooled to provide more efficient estimates.

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May 1991

HOW FAST DO OLD MEN SLOW DOWN?

by

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I. Introduction

This paper uses data on men's track and field and road racing records by age to estimate the rate at which men slow down with age. Eight track, eight field, and eleven road racing events are considered. The track events range from 100 meters to 10,000 meters, and the road racing events range from 5 kilometers to the marathon. The field events are the high jump, pole vault, long jump, triple jump, shot put (16 pounds), discus throw (2 kilograms), hammer throw (16 pounds), and javelin throw (800 grams).

Sections II - V consider the track and road racing events. Section II discusses the methodology that was followed, and Section III presents the estimation results. Section IV compares the age-factors published in Masters Age-Graded Tables (MAGT) with the age-factors implied by this study. It will be seen that the MAGT age-factors seem to be excessively variable and to be biased against older runners. Table 3 presents the age-factors implied by this study. This table should be of interest to old runners and to race officials organizing age-graded races. Section V provides a brief comparison

[1]I am indebted to Don Andrews, William Brainard, Gregory Chow, Stephen Goldfeld, Ethan Nadel, and Christopher Sims for helpful comments and to Alvin Klevorick for pointing out an error in the data. I am also indebted to Peter Mundle for helpful discussions about the track data and to Basil and Linda Honikman at TACSTATS for supplying me with the road racing data and answering various questions about the data.

of the present results to results in the physiological literature. Section VI presents the results for the field events, and Table 5 presents the age-factors for the field events implied by this study.

II. The Methodology

The Model

For a given track or road racing event, let q denote the log of a runner's time in the race. For all runners of a given age, the theoretical frequency distribution for q probably looks something like that depicted in Figure 1. b, the lower bound, is the fastest time that could ever be run by a man of that age. Think of b as the biological limit of men, given perfect race conditions (but no tail winds allowed) and the use of the best training methods and equipment possible (but no performance enhancing drugs allowed). m is the median of the distribution, and u is the upper bound.[2]

This paper focuses on b. Let b_k denote the lower bound for runners of age k. One would expect b_k when plotted against k to look something like that depicted in Figure 2. (Remember that times are measured in logs, so the rates of change are percentage rates of change.) b_k is infinite for small babies, falls to some minimum at age k_1, stays at this minimum to age k_2, and then begins to rise. After b_k begins to rise, one would expect that the rate of slowing down is fairly constant through a certain age k_3 and then begins to rise. k_4 in the figure is the oldest age at which anyone could finish the race.

[2] If runners are included in the population who do not finish the race, then u might be considered to be infinite. This paper does not use u in the analysis, and so it does not matter for present purposes what is assumed about u.

FIGURE 1

Theoretical Frequency Distribution for q.

q = log of time

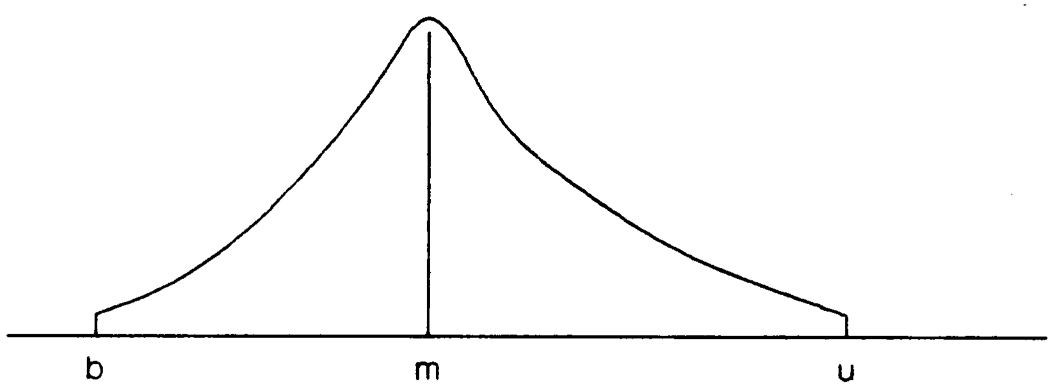
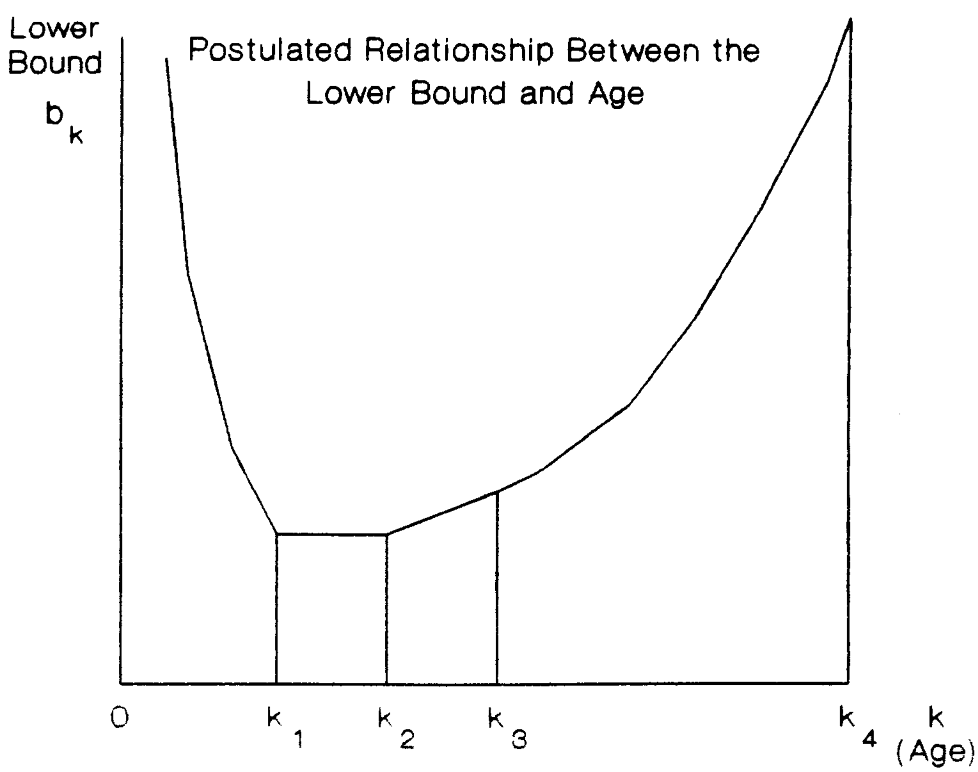


FIGURE 2

Postulated Relationship Between the Lower Bound and Age



The purpose of this paper is to estimate the function in Figure 2 from sometime after age k_2 on. The starting age used in the empirical work is 35, which means that k_2 is assumed to be less than or equal to 35. k_2 need not be equal to 35. If it is not, this just means that the sample used in this paper picks up the line sometime after k_2.

The functional form in Figure 2 is assumed in the empirical work to be linear between k_2 and k_3 and quadratic after that. At k_3, the linear and quadratic curves are assumed to touch and to have the same first derivative. The specification is:

$$(1)\quad b_k = \begin{cases} \alpha_1 + \alpha_2 k & \text{for } k_2 \le k \le k_3 \\ \alpha_3 + \alpha_4 k + \alpha_5 k^2 & \text{for } k > k_3 \end{cases},$$

with the restrictions

$$(2)\quad \begin{aligned} \alpha_3 &= \alpha_1 + \alpha_5 k_3^2 \\ \alpha_4 &= \alpha_2 - 2\alpha_5 k_3 \end{aligned}.$$

The two restrictions force the curves to touch and to have the same first derivative at k_3.[3] The unrestricted parameters to be estimated are α_1, α_2, α_5, and k_3.

The Data

The track data are from Masters Age Records For 1990, and the road racing data are from TACSTATS/USA. The track data give the current world record by age for each event. The road racing data give the current best time

[3]These restrictions are examples of polynomial spline restrictions. See Poirier (1976) for a general discussion of polynomial splines.

by an American for each event (data on world records by age are not yet available for road racing). Let r_k denote the log of the observed record time for age k for a given event, and let ϵ_k denote the difference between r_k and the unobserved b_k. r_k can thus be written:

$$(3) \quad r_k = b_k + \epsilon_k \ .$$

ϵ_k is the measurement error for r_k.

In principle ϵ_k can be either negative or positive, although negative measurement error does not seem likely. Two possible reasons for negative measurement error are 1) the true distance of the race is shorter than the stated distance and 2) the time is recorded wrong in favor of the runner. These kinds of errors are likely to be small because the races and records are monitored closely.

The story is different, however, regarding positive measurement error. The relevant question to consider is how many races for a given event have to be run by runners of age k before r_k becomes a good estimate of b_k? Let N_k denote the (unobserved) number of men age k who have run the particular event in question up to the current time. If N_k is in the millions, as it may be for runners in their 30's and 40's, there is probably a good chance that one has sampled close to the theoretical lower bound. If, on the other hand, N_k is only in the thousands or tens of thousands, as it probably is for very old runners, one is not likely to have sampled close to the lower bound. In fact, it is commonly stated that there are now many more runners, say, age 50 than there used to be, and as these runners age, the age records are likely to fall considerably. In 1989, nine age records in the 100 meters were broken, six of these for ages over 80. Eleven age records in the 10,000 meters were broken, seven of these for ages over 60. Results for other events are similar.[4] The large number of records broken in a single year indicates that the lower bound is far from being observed for many ages. This problem of not having a large enough sample at the higher ages to get a good estimate of the lower bound will be called the "small N_k" problem.[5]

Two adjustments were made in the data to try to account for the small N_k problem. First, the above theory postulates that after age k_2, b_k is greater than b_{k-i} for i positive (men slow down with age). Therefore, if r_k is greater than r_{k+i} for any positive i, r_k must have a relatively large positive measurement error associated with it. Observations of this kind, where the time for a given age is greater than the time for some older age, were not used.

Second, observations at very high ages were not used. The ages not used were always over 78 and in most cases over 81. The highest age used was 89, for 100 meters. The age cutoffs were chosen at the point where there was a large increase in the record time from one age to the next relative to the sizes of the previous increases. In discarding these observations it is implicitly assumed that the slow times are due to the small N_k problem and not to the fact that there is actually a large jump at that age. In other words, the problem is assumed to be a sampling problem, not a biological characteristic.

[4]Compare the records in Masters Age Records 1990 with those in Masters Age Records 1989.

[5]The reason women were not considered in this study is that the small N_k problem seems very serious for them. Societies have not generally encouraged old women to run in track meets and road races.

These two adjustments may not be enough to completely eliminate the small N_k problem, and so the following results may be biased in the sense of overestimating the slowdown rate, especially at the older ages. An interesting question for future research is whether more can be done with the current data to try to adjust for the small N_k problem. It is the case, for example, that N_k is likely to be a decreasing function of k and that ϵ_k is a decreasing function of N_k. Therefore, ϵ_k is likely to be an increasing function of k. The approach taken in this study in dealing with this problem is simply to truncate the sample at the point where the size of the effect of k on ϵ_k appears to become large. An alternative approach would be to parameterize the function relating k to ϵ_k (say $\epsilon_k = \gamma_1 + \gamma_2 k + \gamma_3 k^2$ for k greater than some value $\bar{k}$), add this to (3), and try to estimate the new parameters (γ_1, γ_2, γ_3, $\bar{k}$) along with the others. My feeling is that the data are not good enough to allow anything sensible to come out of this, but it may be worth further thought.

Another possible approach is the following. Denote the density function in Figure 1 for a given age k as $f(q_k,\theta_k)$, where q_k is the log of the time in the event of an individual of age k and θ_k is a vector of parameters. Let q_k^{min} denote the minimum value of q_k in a sample of size N_k. q_k^{min} is an order statistic, and let $g(q_k^{min},\theta_k,N_k)$ denote its density function. The functional form of g depends, of course, on the functional form of f. The data used in this study are observations on q_k^{min} for k 35 and over. Given 1) observations on q_k^{min} , 2) an assumption about the functional form of f, 3) a parameterization of the elements of θ_k as functions of k, and 4) values for N_k or a parameterization of N_k as a function of k, one could estimate the parameters by maximum likelihood. Again, I doubt that the data are good enough to allow sensible estimates to be obtained using this approach, but it may be worth a try.

Until further work is done, the present results should be interpreted with caution. If the same estimation is done ten or twenty years hence, it is likely that the estimated slowdown rates will be smaller than the currently estimated rates. Whether they will be only slightly smaller or a lot smaller is the key open question.

Note finally that if all ages are getting better over time (say because of better nutrition, better training methods, or better equipment), this will not affect the estimated slowdown rates as long as all ages are getting better at the same rate. Progress like this will affect the estimated slowdown rates only if it differently affects the various ages.

The Econometrics

Let $d_k = 1$ if $k \leq k_3$ and $d_k = 0$ if $k > k_3$. Using this notation, substituting (1) into (3), and using the restrictions in (2) yields the equation to be estimated:

$$(4) \quad r_k = \alpha_1 + \alpha_2 k + \alpha_5(1-d_k)(k_3^2 - 2k_3 k + k^2) + \epsilon_k \quad , \; k = 35,\ldots K \;.$$

There are four parameters to estimate, α_1, α_2, α_5, and k_3, where it should be remembered that d_k is a function of k_3. K is the oldest age in the sample period. There are age gaps in the sample period because of the exclusion of observations with dominated times.

The main interest in this study is in the derivative of r_k with respect to k. This derivative is:

$$(5) \quad \partial r_k/\partial k = \alpha_2 + 2\alpha_5(1-d_k)(k - k_3) \quad .$$

This derivative is not a function of the constant term α_1 in (4), and so the size of the constant term is not of direct concern here.

Equation (4) pertains to a particular event. If one is willing to assume that α_2, α_5, and k_3 are the same across events, then the data on the different events can be pooled and more efficient estimates obtained. It does not seem unreasonable that the derivatives are the same at least for events close to each other in distance. When the data are pooled, different constant terms are needed for each event, since these obviously vary with distance. When the data were pooled for the results below, the following equation was estimated (n is the number of events pooled):

$$(6)\quad r_{ik} = \beta_1 D_{1ik} + \ldots + \beta_n D_{nik} + \alpha_2 k + \alpha_5(1-d_{ik})(k_3^2 - 2k_3 k + k^2) + \epsilon_{ik}\ ,$$
$$i = 1,\ldots,n\ ;\ k = 35,\ldots,K_i\ ,$$

where r_{ik} is the log of the observed record for event i and age k, D_{jik} is a dummy variable that is equal to one when event i is equal to j and zero otherwise ($j = 1,\ldots,n$), $d_{ik} = 1$ if $k \le k_3$ and $d_{ik} = 0$ if $k > k_3$, ϵ_{ik} is the measurement error for event i and age k, and K_i is the oldest age used for event i. Again, there are age gaps in the sample period for a given event because of the exclusion of dominated observations. The n β_i coefficients in equation (6) pick up the n different constant terms.

Return now to the estimation of equation (4). Since positive measurement error for r_k is more likely than negative measurement error, the mean of ϵ_k is likely to be positive. If there is no negative measurement error at all, then $\epsilon_k \ge 0$ for all k. A positive mean for ϵ_k poses no problem in the estimation of equation (4) because the positive mean is merely absorbed in the estimate of the constant term. If the mean of ϵ_k is $\bar{\epsilon}$, define $\epsilon_k^* =$

$\epsilon_k - \bar{\epsilon}$, where ϵ_k^* has mean zero. Equation (4) can then be rewritten with ϵ_k^* replacing ϵ_k and the constant term changed from α_1 to $\alpha_1 + \bar{\epsilon}$. In this case α_1 is not identified, but this is of no concern here because the derivatives do not depend on α_1. One can thus estimate (4) by nonlinear least squares in the usual way. This estimation procedure will be called the NLS procedure.

There is, however, another estimation method that is of interest to consider. Under the assumption that $\epsilon_k \geq 0$ for all k, equation (4) can be estimated under the restriction that all estimated residuals are non negative. This procedure is common in the estimation of frontier production functions -- see, for example, Aigner and Chu (1968) and Schmidt (1976). The one added complication here is that equation (4) is nonlinear in coefficients. For linear equations the estimation problem can be set up as a quadratic programming problem and solved by standard methods, but for nonlinear equations some other procedure must be found.

The procedure used for the results below is the following. In the standard case the coefficients in equation (4) are estimated by minimizing the sum of squared residuals $\Sigma_{k=35}^{K}\hat{\epsilon}_k^2$. Instead, one can minimize a weighted sum $\Sigma_{k=35}^{K}\lambda_k\hat{\epsilon}_k^2$, where λ_k is equal to 1 if $\hat{\epsilon}_k \geq 0$ and is equal to a number greater than one if $\hat{\epsilon}_k < 0$. This penalizes negative errors more than non negative ones. For the work below a value of 100 was used for λ_k when $\hat{\epsilon}_k$ was less than zero. This was large enough to make nearly all the estimated residuals non negative at the optimum.[6] This estimation procedure will be called the "frontier" procedure.

[6]This procedure does not guarantee that all the estimated residuals are non negative because if $\hat{\epsilon}_k$ is negative but very close to zero, its contribution to the objective function is small even if λ_k is large. In practice the negative errors were very close to zero and were for all intents and purposes zero.

It turns out, as will be seen below, that the use of the frontier procedure over the NLS procedure has only a small effect on the estimated slope coefficients and thus on the estimated derivatives. The use of the frontier procedure primarily affects the estimate of the constant term, which is not of concern here.

An attempt was also made to estimate the parameters of (4) under the assumption that ϵ_k follows a gamma distribution, as discussed in Greene (1980). The use of this distribution has the advantage of allowing the statistical properties of the maximum likelihood estimator to be readily obtained, which the procedure discussed above does not. It also accommodates quite flexible shapes of the error distributions. Unfortunately, sensible results could not be obtained following this approach. The estimates of the two distribution parameters (P and λ in Greene's (1980) notation) were usually not sensible, and convergence was hard to obtain. It would be interesting to see in future work if this approach could be made to work, but the effort so far (which was considerable) was not successful.

III. The Results

The results of estimating the equation for each event by itself are presented first in Table 1. The estimates of α_2, k_3, and α_5 and their estimated standard errors are presented along with the implied values of the derivatives at ages 50, 60, 75, and 95. (The implied value of the derivative for ages below $\hat{k}_3$ is $\hat{\alpha}_2$.) The estimation technique for these results is NLS.

Set aside for the moment the 100 meter, 200 meter, and marathon events. Of the remaining 14 events, two stand out as being considerably different from the rest in Table 1: 10,000 meters and 5K. For 10,000 meters there is a small

TABLE 1

The Estimation Results for the Track and Road Racing Events

Event		$\hat{\alpha}_2$	$SE(\hat{\alpha}_2)$	$\hat{k}_3$	$SE(\hat{k}_3)$	$\hat{\alpha}_5$	$SE(\hat{\alpha}_5)$	Derivative at age 50	60	75	95	SE	No. Obs.	Max Age
Track (meters)														
1	100	.0048	.0013	46.5	7.9	.00013	.00002	.0057	.0083	.0123	.0175	.015	29	89
2	200	.0076	.0003	65.8	1.8	.00056	.00012	.0076	.0076	.0179	.0403	.012	27	82
3	400	.0068	.0012	51.0	5.5	.00021	.00004	.0068	.0106	.0168	.0251	.016	25	81
4	800	.0052	.0083	37.1	37.2	.00012	.00002	.0083	.0107	.0143	.0192	.014	28	79
5	1500	.0088	.0006	54.0	4.4	.00018	.00004	.0088	.0109	.0162	.0233	.013	31	82
6	3000	.0080	.0008	52.8	4.1	.00024	.00005	.0080	.0114	.0186	.0281	.015	24	82
7	5000	.0087	.0009	50.1	6.3	.00013	.00003	.0087	.0112	.0150	.0201	.013	29	83
8	10000	.0089	.0014	47.3	17.8	.00006	.00004	.0092	.0105	.0125	.0150	.015	24	82
Road Racing														
9	5K	.0075	.0012	57.6	6.2	.00035	.00013	.0075	.0091	.0195	.0335	.027	24	82
10	10K	.0071	.0009	51.9	5.7	.00020	.00006	.0071	.0104	.0164	.0244	.020	31	81
11	15K	.0066	.0013	48.7	7.5	.00016	.00004	.0070	.0101	.0147	.0210	.016	28	82
12	10MI	.0066	.0031	45.1	19.6	.00011	.00005	.0077	.0100	.0134	.0180	.023	20	81
13	20K	.0052	.0044	42.2	21.9	.00014	.00005	.0073	.0100	.0141	.0196	.025	18	81
14	1/2MA	.0042	.0076	40.1	25.0	.00018	.00004	.0077	.0113	.0166	.0237	.029	22	81
15	30K	.0054	.0033	46.7	13.5	.00023	.00012	.0070	.0116	.0185	.0277	.032	12	78
16	20MI	.0055	.0024	49.1	9.4	.00028	.00011	.0059	.0115	.0198	.0309	.027	11	78
17	MA	.0063	.0009	58.2	2.5	.00061	.00012	.0063	.0085	.0269	.0515	.019	21	79
Pooled														
18[a]		.0069	.0006	47.7	3.0	.00016	.00001	.0076	.0109	.0157	.0221	.021	256	83
19[b]		.0057	.0018	49.0	7.5	.00026	.00009	.0062	.0115	.0194	.0299	.030	23	78
Frontier Method														
	Line[c]													
20	1	.0046	-	49.3	-	.00014	-	.0048	.0077	.0120	.0177	-	29	89
21	2	.0079	-	66.9	-	.00064	-	.0079	.0079	.0183	.0441	-	27	82
22	18	.0081	-	51.4	-	.00014	-	.0081	.0104	.0146	.0201	-	256	83
23	19	.0045	-	51.8	-	.00035	-	.0045	.0103	.0209	.0351	-	23	78
24	17	.0053	-	59.0	-	.00077	-	.0053	.0068	.0375	.0606	-	21	79

Notes:

[a]The pooled equations are 1-7,10-14.
[b]The pooled equations are 14,15.
[c]The frontier method used for the equation in this line above.

Max Age - Oldest age used in the sample period.

estimate of α_5, which means that the derivatives grow very slowly with age. For 5K the opposite is true. Note in particular that 10,000 meters is quite different from 10K even though it is the same distance, and likewise for 5K and 5,000 meters. I am inclined to discount the 10,000 meter and 5K results as likely reflecting considerable measurement error, given that they are so different from the rest.

The other two events that have somewhat different results are 30K and 20 miles. These both have slightly larger estimates of α_5 than the others. Two things could be going on here. First, it may be that at roughly the 30K distance, the slowdown rate at a given age begins to increase, and this is what the estimates are picking up. Second, the results may be unreliable. The 30K and 20 mile events are not as popular as the others, and so there is more of a potential small N_k problem here. The potential small N_k problem also reveals itself in the fact that the samples are small for these two events (12 and 11 observations respectively). The samples are small because many of the records were dominated by records at older ages and so were discarded. The high number of dominated records probably indicates a small N_k problem. It is thus an open question as to whether the 30K and 20 mile results are picking up an increase in the slowdown rate at a given age across distances or are simple due to a small sample problem.

The remaining five track events (400 meters through 5,000 meters) and five road racing events (10K through the half marathon) give similar results. There is no evidence of anything varying in a systematic way across distances. The implied derivatives at age 60 across the ten distances are in remarkably close agreement; the range is only .0100 at 10 miles and 20K to .0114 at 3000 meters. There is more variation in the estimates of α_2, where the range is

.0042 at the half marathon to .0087 at 5000 meters. The range at age 75 is .0134 at 10 miles to .0186 at 3000 meters, and the range at age 95 is .0180 at 10 miles to .0281 at 3000 meters. The estimated standard errors for $\hat{\alpha}_2$ and $\hat{k}_3$ are fairly large for some events.

Given that no systematic variation across distances is evident in the ten events, it seems sensible to pool them to obtain more efficient estimates. The results of doing this are reported in line 18 in Table 1. The estimate of α_2 is .0069, with an estimated standard error of .0006, and the estimate of k_3 is 47.7, with an estimated standard error of 3.0. The derivatives are .0076 at age 50, .0109 at age 60, .0157 at age 75, and .0221 at age 95.[7]

These pooling results are not sensitive to the exclusion of the 10,000 meters, 5K, 30K, and 20 mile events. When the observations from these events are included in the pooling, the estimates of α_2 and k_3 are .0069 and 48.3 respectively, and the derivatives at ages 50, 60, 75, and 95 are .0075, .0109, .0159, and .0227 respectively. These estimates are very close to the estimates presented in Table 1 when the four events are excluded.

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[7]Under the assumption that ϵ_k is normally distributed, which cannot be quite right because of the truncation issues, an F test can be used to test the hypothesis that α_2, α_5, and k_3 are the same across the ten events. There are 27 restrictions, and the number of observations in the pooled regression is 256. The F-value was 2.17, which compares with the critical value at the 1 percent level of 1.82, and so the hypothesis is rejected. Similar results were obtained when other sets of events were used. The hypothesis that the coefficients are the same across the specified events was usually rejected, although the computed F-values were usually not too much above the critical values. (The hypothesis that the coefficients are the same for 30K and 20 miles was, however, not rejected at the 5 percent level.)

I am not inclined to take these rejections as evidence against pooling. The computed F values were never too far from not rejecting the null hypothesis; the sample size is small relative to the number of restrictions; and there seems to be no compelling reason for believing that the coefficients change across the particular events, especially since no systematic patterns across the ten events were evident when the equations were estimated individually.

Consider now the other events. For 100 meters the results indicate that the rate of slowdown is smaller than it is for the other events. The estimated age at which the quadratic takes over is similar for 100 meters versus the pooled sample (46.5 versus 47.7), but the size of the derivatives are smaller. For example, at age 60 the slowdown rate is .0083 compared to .0109 for the pooled sample. At age 95 the rate is .0175 compared to .0221 for the pooled sample.

The results for 200 meters are quite different from the rest. The estimated age at which the quadratic takes over is 65.8, which is much larger than the other estimates. Also, the estimate of α_5 is much larger, which means that once the quadratic takes over, the estimated increase in the slowdown rate is larger than it is for the other events. The derivatives at age 60 are similar for 100 and 200 meters, but the derivative is noticeably larger at age 75 for 200 meters and considerably larger at age 95 (.0403 versus .0175). Because the 200 meter results stand out as being much different from the rest -- both from the 100 meter results and from the results for 400 meters and above -- they should be interpreted with considerable caution. It seems likely, for example, that the increase in the slowdown rate after age 64 has been overestimated.

Given that the results for 30K and 20 miles are similar to each other and differ somewhat from the rest, it is of interest to pool the two events. The results of this pooling are presented in line 19 in Table 1. Comparing lines 18 and 19, it can be seen that the estimated slowdown rate for 30K and 20 miles is less at the younger ages and more at the older ages. Although not shown in the table, the age at which the slowdown rate becomes greater for 30K and 20 miles is about 59. By age 95 the estimated slowdown rate is .0299 for

30K and 20 miles versus .0221 for the others.

The results for the marathon in line 17 continue the pattern of the estimated slowdown rate being less at the younger ages and more at the older ages. Although not shown in the table, the age at which the slowdown rate becomes greater for the marathon compared to the pooled events in line 18 is about 63. The estimated age at which the quadratic takes over is 58.2, which is higher than all the other estimates except the one for 200 meters. The estimate of α_2 is .0063, which means that until age 58 the estimated slowdown rate is constant at .63 percent per year. After age 58 the estimated slowdown rate picks up fairly rapidly (the estimate of α_5 is large), and by age 95 the derivative is by far the largest of any event at .0515. This derivative is even much larger than the derivative for the 30K and 20 mile events.

The differences between the marathon derivatives and the other derivatives at the older ages are large enough to make one question whether the marathon results should be trusted. There may be, however, more to the marathon than a mere 6.2 miles beyond 20 miles. Anyone who has run the last 6.2 miles in a marathon can appreciate this. If there is an important nonlinearity in going from 20 miles to the marathon, one might expect there to be a more rapid increase in the rate of slowing down at older ages for the marathon. This is what the current results show, although the estimated size of the effects should be taken with considerable caution.

The final estimates in Table 1 were obtained using the frontier procedure. Results are presented for 100 meters, 200 meters, pooled 400 meters through the half marathon, pooled 30K and 20 miles, and the marathon. The results using the frontier procedure are quite similar to the other results. None of the comparisons and conclusions discussed above are changed by the frontier results. Figure 3 shows a plot of the actual and predicted values for the marathon equation using the frontier procedure. The plots for the other events are similar in that the actual values are always close to or greater than the values on the frontier.

Finally, it should be mentioned that two other functional forms were tried. First, the quadratic in (1) was replaced with $b_k = \alpha_3 + \alpha_4/(k-\alpha_5)$ for $k > k_3$. The use of this form did not generally lead to as good fits as did the quadratic, and the curvature seemed too extreme at the top ages. Second, the quadratic was made more general by replacing the exponent 2 with a coefficient (α_6) to be estimated: $b_k = \alpha_3 + \alpha_4 k + \alpha_5 k^{\alpha_6}$. This allows the curvature to be either more or less extreme than that implied by the quadratic. This did not work because the estimates of α_5 and α_6 were too collinear for any confidence to be placed on the results. The estimates of α_6 were generally around 2, with large estimated standard errors.

IV. Age-Graded Tables

It is possible to use the coefficient estimates in Table 1 to estimate the ratio (denoted R_k) of the lower bound time at a given age k to the best lower bound time regardless of age. To do this, one needs a starting point, which in the present case is a value for R_{35}. Given R_{35}, R_{36} is $R_{35}(1+D_{36})$, where D_k is the derivative at age k computed from the estimated equation (remember that the derivatives are in percentage terms). R_{37} is then $R_{36}(1+D_{37})$, and so on.

The inverse of R_k is called an "age-factor" in Masters Age-Graded Tables (MAGT), and tables of age-factors are presented in MAGT for various events. Although MAGT does not explain how the age-factors were arrived at, it is of interest to see how they compare to the age-factors computed in this study.

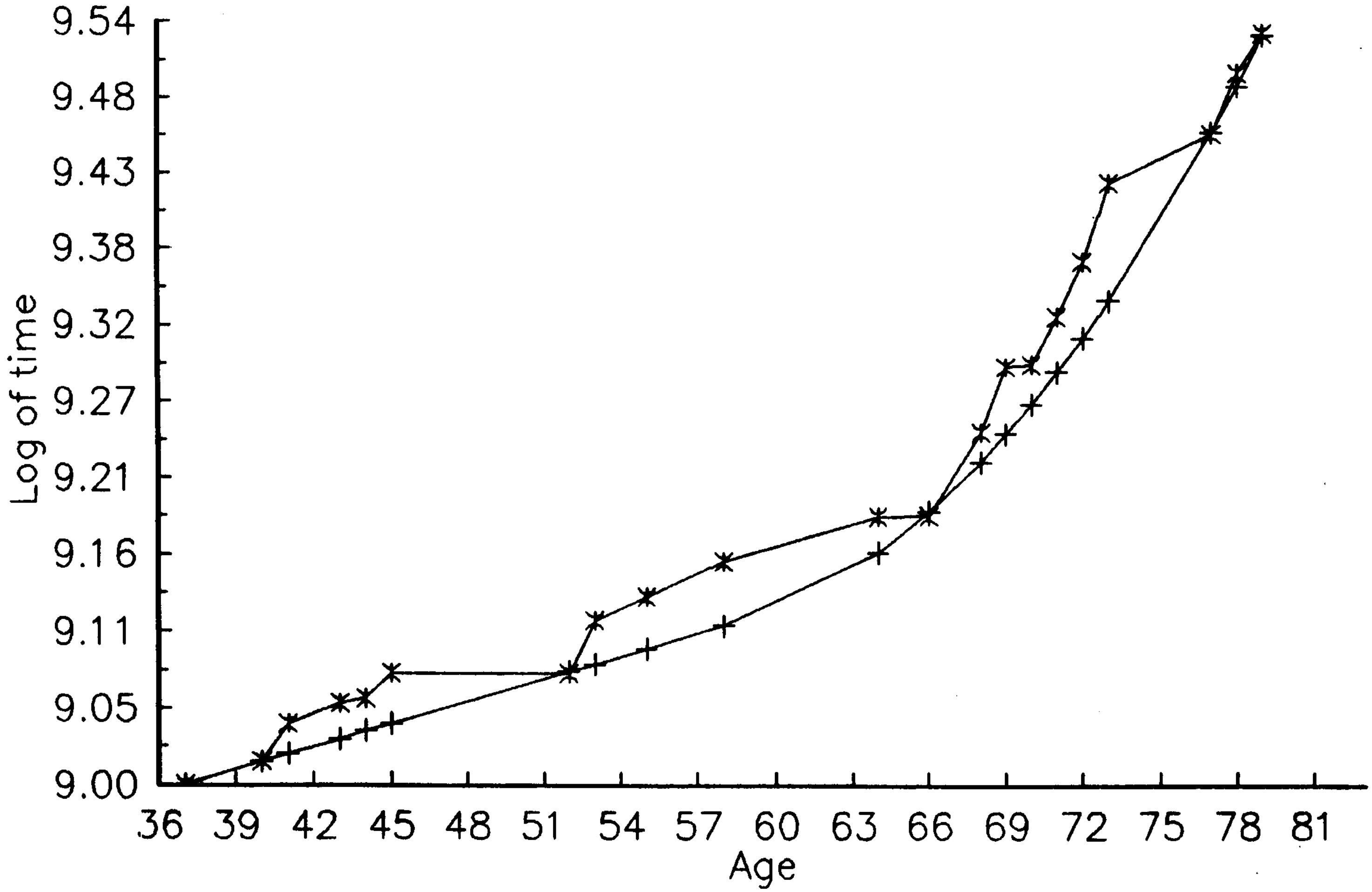

Figure 3
Actual (*) and Predicted (+) Values for the Marathon

Table 2 presents the implied values of R_k (the inverse of the age-factors) from the table on page 24 in MAGT. These age-factors are for the 5K through half marathon events. The percentage changes in R_k are also presented in Table 2, along with the change in the percentage changes. These are the equivalent of the first and second derivatives of equation (4).

Table 2 shows that the MAGT value of R_k for age 35 is 1.02838. This means that MAGT has assumed that some loss in time has occurred by age 35 (2.838 percent to be exact).

Table 2 also presents values of R_k implied by the estimates in line 22 in Table 1. These are the estimates for the pooled events 400 meters through the half marathon, estimated by the frontier procedure. To make these values of R_k comparable to the MAGT values, the MAGT value of 1.02838 was used for R_{35} for the starting point. The derivatives from the equation and the changes in the derivatives are also presented in Table 2.

Two interesting conclusions emerge from in Table 2. First, the MAGT derivatives are (with one exception) increasing with age, but the sizes of the increases are erratic. The derivatives from this study, on the other hand, are constant through age 47 (actually 47.7) and then change at a constant amount (.00028) after that. This constant change is, of course, due to the use of the quadratic functional form. The erratic behavior of the change in the MAGT derivatives does not seem sensible. It seems unlikely, for example, that the derivative would change by .00033 at age 77, .00003 at age 78, .00050 at age 79, and .00004 at age 80. Nature is not generally like this.

The second conclusion is that the values of R_k are always higher for the present study. By age 90 the value of R_k is about 7 percent higher than the MAGT value. MAGT thus assumes that men slow down at a slower rate than seems

TABLE 2

Comparison of Estimated Age-Factors

	MAGT			Present Study		
Age	R_k	D_k	D_k-D_{k-1}	R_k	D_k	D_k-D_{k-1}
35	1.02838	-	-	1.02838	-	-
36	1.03455	0.00600	-	1.03666	0.00805	-
37	1.04080	0.00604	0.00004	1.04501	0.00805	0.00000
38	1.04723	0.00618	0.00014	1.05343	0.00805	0.00000
39	1.05374	0.00622	0.00004	1.06191	0.00805	0.00000
40	1.06045	0.00636	0.00015	1.07047	0.00805	0.00000
41	1.06724	0.00640	0.00004	1.07909	0.00805	0.00000
42	1.07411	0.00644	0.00004	1.08778	0.00805	0.00000
43	1.08120	0.00660	0.00015	1.09654	0.00805	0.00000
44	1.08849	0.00675	0.00015	1.10537	0.00805	0.00000
45	1.09589	0.00679	0.00005	1.11428	0.00805	0.00000
46	1.10327	0.00673	-0.00006	1.12325	0.00805	0.00000
47	1.11086	0.00689	0.00016	1.13230	0.00805	0.00000
48	1.11882	0.00716	0.00027	1.14142	0.00805	0.00000
49	1.12714	0.00744	0.00028	1.15061	0.00805	0.00000
50	1.13585	0.00772	0.00028	1.15988	0.00805	0.00000
51	1.14482	0.00790	0.00018	1.16922	0.00805	0.00000
52	1.15420	0.00819	0.00030	1.17884	0.00823	0.00017
53	1.16401	0.00850	0.00030	1.18887	0.00850	0.00028
54	1.17412	0.00869	0.00019	1.19930	0.00878	0.00028
55	1.18469	0.00900	0.00032	1.21017	0.00906	0.00028
56	1.19589	0.00945	0.00044	1.22146	0.00933	0.00028
57	1.20744	0.00966	0.00021	1.23320	0.00961	0.00028
58	1.21936	0.00988	0.00022	1.24539	0.00989	0.00028
59	1.23153	0.00998	0.00010	1.25804	0.01016	0.00028
60	1.24409	0.01020	0.00023	1.27117	0.01044	0.00028
61	1.25691	0.01031	0.00011	1.28479	0.01071	0.00028
62	1.27000	0.01041	0.00011	1.29891	0.01099	0.00028
63	1.28370	0.01078	0.00037	1.31354	0.01127	0.00028
64	1.29769	0.01090	0.00012	1.32871	0.01154	0.00028
65	1.31199	0.01102	0.00012	1.34441	0.01182	0.00028
66	1.32679	0.01128	0.00026	1.36067	0.01209	0.00028
67	1.34210	0.01154	0.00026	1.37750	0.01237	0.00028
68	1.35777	0.01168	0.00013	1.39492	0.01265	0.00028
69	1.37382	0.01181	0.00014	1.41295	0.01292	0.00028
70	1.39043	0.01210	0.00028	1.43160	0.01320	0.00028
71	1.40726	0.01210	0.00001	1.45090	0.01348	0.00028
72	1.42470	0.01239	0.00029	1.47085	0.01375	0.00028
73	1.44259	0.01255	0.00016	1.49148	0.01403	0.00028
74	1.46113	0.01286	0.00031	1.51282	0.01430	0.00028
75	1.47995	0.01288	0.00002	1.53488	0.01458	0.00028
76	1.49925	0.01304	0.00017	1.55768	0.01486	0.00028
77	1.51930	0.01337	0.00033	1.58125	0.01513	0.00028
78	1.53965	0.01339	0.00003	1.60562	0.01541	0.00028
79	1.56104	0.01389	0.00050	1.63081	0.01569	0.00028
80	1.58278	0.01393	0.00004	1.65684	0.01596	0.00028

TABLE 2 (continued)

	MAGT			Present Study		
Age	R_k	D_k	D_k-D_{k-1}	R_k	D_k	D_k-D_{k-1}
81	1.60514	0.01413	0.00020	1.68374	0.01624	0.00028
82	1.62840	0.01449	0.00037	1.71155	0.01651	0.00028
83	1.65235	0.01471	0.00021	1.74029	0.01679	0.00028
84	1.67701	0.01493	0.00022	1.76999	0.01707	0.00028
85	1.70242	0.01515	0.00023	1.80069	0.01734	0.00028
86	1.72861	0.01538	0.00023	1.83241	0.01762	0.00028
87	1.75593	0.01580	0.00042	1.86521	0.01790	0.00028
88	1.78380	0.01588	0.00007	1.89910	0.01817	0.00028
89	1.81291	0.01632	0.00044	1.93414	0.01845	0.00028
90	1.84264	0.01640	0.00008	1.97035	0.01872	0.00028

Notes:

MAGT = Masters Age-Graded Tables

R_k is the inverse of the age-factors in MAGT.

The MAGT age-factors are for the 5K through half marathon events.

The age-factors from the present study are from the estimates in line 22 in Table 1.

D_k is the percentage change in R_k: $D_k = (R_k/R_{k-1}) - 1$.

warranted by the data.

Table 3 presents five sets of values of R_k implied by the present study. The values are based on the coefficient estimates in lines 20-24 in Table 1, which were obtained using the frontier estimation procedure. The starting values of R_k (at age 35) are taken from MAGT. The values of R_k are presented through age 100, although the values for about age 83 and above are extrapolations beyond the end of the estimation period and should be interpreted with more caution.

As noted in Section III, the 200 meter results are somewhat suspect. If the 200 meter results are ignored, Table 3 shows that beginning with age 79 the values of R_k increase with the length of the race. At age 90 the values are 1.6979 for 100 meters, 1.9704 for 400 meters through the half marathon, 2.2169 for 30K and 20 miles, and 2.8573 for the marathon. If the best marathon time is taken to be 2 hours and 6 minutes, the value of R_k for the marathon implies that the best time for a 90 year old is 6 hours (2.8573 times 2 hours and 6 minutes). At age 100 the four values of R_k are, respectively, 2.0265, 2.4076, 3.1398, and 5.1821, although again these values are extrapolations way beyond the end of the estimation period.

Coming back to the MAGT values, although not shown in Table 2, the MAGT value of R_k at age 90 for 100 meters is 1.6736. This is again lower than the value of 1.6979 in Table 3, although in this case the values are quite close. The MAGT value of R_k at age 90 for the marathon is 1.8171, which is considerably lower than the value of 2.8573 in Table 3. The MAGT values imply that the slowdown rates for the marathon are smaller than they are for the 5K through half marathon events, which is opposite from what the empirical results seem to show and from what is presented in Table 3. Using a best marathon time of 2 hours and 6 minutes, the MAGT value of 1.8171 for age 90 implies that the best time for a 90 year old is 3 hours and 49 minutes, which compares to the above estimate of 6 hours using the results in this study.

Although the focus of this paper is on record times, Table 3 can be used by individual runners to estimate their age-adjusted times if one additional assumption is made. If in Figure 1 the difference between one's position on the horizontal axis and b does not change as b changes, then the results in Table 3 can be used. If this is true, it simply means that one's times are increasing at the same percentage rate as the record times are increasing. Obviously, injury or illness will increase one's distance from b. Also, if average runners slow down at a different rate from elite runners, then the distance from b for an average runner will be changing over time, thus making the results in Table 3 unreliable. Finally, if prolonged running wears parts of the body out -- the opposite situation from use-it-or-lose-it -- then one's distance from b will change over time as a function of how much past running has been done. This will also make the results in Table 3 unreliable.

Given the assumption that one's distance from b is constant over time and given an estimate of one's best time ever in the event, the values of R_k in Table 3 can be used to compute one's projected times by age. Race officials can also use the values to adjust each runner's time for his age.

V. Comparison to the VO_{2max} Results

A common measure of aerobic capacity in physiology is VO_{2max}. It is well known that VO_{2max} declines with age, and it is of interest to see how this decline compares to the decline in running performance estimated in this study. There seems to be nothing in the physiological literature for VO_{2max}

TABLE 3

Estimated Age Factors

R_k - Projected time for age k divided by overall best time
D_k - Percentage change in R_k

	100 meters		200 meters		400 meters - Half Marathon		30K, 20 miles		Marathon	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
35	1.0368	0.0046	1.0442	0.0068	1.0284	0.0080	1.0284	0.0045	1.0143	0.0053
36	1.0416	0.0046	1.0512	0.0068	1.0367	0.0080	1.0330	0.0045	1.0197	0.0053
37	1.0463	0.0046	1.0583	0.0068	1.0450	0.0080	1.0376	0.0045	1.0251	0.0053
38	1.0512	0.0046	1.0655	0.0068	1.0534	0.0080	1.0422	0.0045	1.0305	0.0053
39	1.0560	0.0046	1.0727	0.0068	1.0619	0.0080	1.0469	0.0045	1.0360	0.0053
40	1.0608	0.0046	1.0800	0.0068	1.0705	0.0080	1.0515	0.0045	1.0415	0.0053
41	1.0657	0.0046	1.0873	0.0068	1.0791	0.0080	1.0562	0.0045	1.0470	0.0053
42	1.0706	0.0046	1.0946	0.0068	1.0878	0.0080	1.0610	0.0045	1.0526	0.0053
43	1.0755	0.0046	1.1020	0.0068	1.0965	0.0080	1.0657	0.0045	1.0581	0.0053
44	1.0804	0.0046	1.1095	0.0068	1.1054	0.0080	1.0704	0.0045	1.0637	0.0053
45	1.0854	0.0046	1.1170	0.0068	1.1143	0.0080	1.0752	0.0045	1.0694	0.0053
46	1.0904	0.0046	1.1245	0.0068	1.1233	0.0080	1.0800	0.0045	1.0751	0.0053
47	1.0954	0.0046	1.1321	0.0068	1.1323	0.0080	1.0849	0.0045	1.0808	0.0053
48	1.1004	0.0046	1.1398	0.0068	1.1414	0.0080	1.0897	0.0045	1.0865	0.0053
49	1.1055	0.0046	1.1475	0.0068	1.1506	0.0080	1.0946	0.0045	1.0922	0.0053
50	1.1107	0.0048	1.1553	0.0068	1.1599	0.0080	1.0994	0.0045	1.0980	0.0053
51	1.1164	0.0051	1.1631	0.0068	1.1692	0.0080	1.1043	0.0045	1.1039	0.0053
52	1.1224	0.0054	1.1709	0.0068	1.1788	0.0082	1.1094	0.0046	1.1097	0.0053
53	1.1287	0.0056	1.1789	0.0068	1.1889	0.0085	1.1153	0.0053	1.1156	0.0053
54	1.1354	0.0059	1.1868	0.0068	1.1993	0.0088	1.1220	0.0060	1.1215	0.0053
55	1.1424	0.0062	1.1949	0.0068	1.2102	0.0091	1.1296	0.0067	1.1275	0.0053
56	1.1499	0.0065	1.2029	0.0068	1.2215	0.0093	1.1380	0.0074	1.1334	0.0053
57	1.1577	0.0068	1.2111	0.0068	1.2332	0.0096	1.1472	0.0081	1.1395	0.0053
58	1.1659	0.0071	1.2193	0.0068	1.2454	0.0099	1.1574	0.0089	1.1455	0.0053
59	1.1745	0.0074	1.2275	0.0068	1.2580	0.0102	1.1684	0.0096	1.1516	0.0053
60	1.1835	0.0077	1.2358	0.0068	1.2712	0.0104	1.1804	0.0103	1.1594	0.0068
61	1.1929	0.0079	1.2442	0.0068	1.2848	0.0107	1.1934	0.0110	1.1690	0.0083
62	1.2027	0.0082	1.2526	0.0068	1.2989	0.0110	1.2073	0.0117	1.1806	0.0099
63	1.2129	0.0085	1.2611	0.0068	1.3135	0.0113	1.2223	0.0124	1.1940	0.0114
64	1.2236	0.0088	1.2709	0.0078	1.3287	0.0115	1.2383	0.0131	1.2094	0.0129
65	1.2347	0.0091	1.2821	0.0088	1.3444	0.0118	1.2554	0.0138	1.2270	0.0145
66	1.2463	0.0094	1.2946	0.0098	1.3607	0.0121	1.2736	0.0145	1.2466	0.0160
67	1.2584	0.0097	1.3085	0.0108	1.3775	0.0124	1.2930	0.0152	1.2685	0.0175
68	1.2709	0.0100	1.3239	0.0118	1.3949	0.0126	1.3136	0.0159	1.2927	0.0191
69	1.2839	0.0102	1.3408	0.0128	1.4129	0.0129	1.3355	0.0166	1.3193	0.0206
70	1.2974	0.0105	1.3593	0.0138	1.4316	0.0132	1.3586	0.0174	1.3486	0.0222
71	1.3115	0.0108	1.3793	0.0147	1.4509	0.0135	1.3832	0.0181	1.3805	0.0237
72	1.3260	0.0111	1.4010	0.0157	1.4708	0.0138	1.4091	0.0188	1.4153	0.0252
73	1.3411	0.0114	1.4244	0.0167	1.4915	0.0140	1.4366	0.0195	1.4532	0.0268
74	1.3568	0.0117	1.4497	0.0177	1.5128	0.0143	1.4656	0.0202	1.4944	0.0283
75	1.3730	0.0120	1.4768	0.0187	1.5349	0.0146	1.4962	0.0209	1.5390	0.0298
76	1.3898	0.0123	1.5060	0.0197	1.5577	0.0149	1.5285	0.0216	1.5872	0.0314

TABLE 3 (continued)

	100 meters		200 meters		400 meters - Half Marathon		30K, 20 miles		Marathon	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
77	1.4072	0.0125	1.5372	0.0207	1.5813	0.0151	1.5626	0.0223	1.6395	0.0329
78	1.4253	0.0128	1.5705	0.0217	1.6056	0.0154	1.5986	0.0230	1.6960	0.0345
79	1.4440	0.0131	1.6062	0.0227	1.6308	0.0157	1.6365	0.0237	1.7570	0.0360
80	1.4633	0.0134	1.6442	0.0237	1.6568	0.0160	1.6765	0.0244	1.8229	0.0375
81	1.4834	0.0137	1.6848	0.0247	1.6837	0.0162	1.7187	0.0251	1.8941	0.0391
82	1.5041	0.0140	1.7281	0.0257	1.7115	0.0165	1.7631	0.0259	1.9710	0.0406
83	1.5255	0.0143	1.7742	0.0267	1.7403	0.0168	1.8099	0.0266	2.0541	0.0421
84	1.5477	0.0146	1.8233	0.0277	1.7700	0.0171	1.8593	0.0273	2.1438	0.0437
85	1.5707	0.0148	1.8756	0.0287	1.8007	0.0173	1.9113	0.0280	2.2407	0.0452
86	1.5944	0.0151	1.9312	0.0297	1.8324	0.0176	1.9662	0.0287	2.3455	0.0467
87	1.6190	0.0154	1.9904	0.0307	1.8652	0.0179	2.0240	0.0294	2.4587	0.0483
88	1.6444	0.0157	2.0534	0.0316	1.8991	0.0182	2.0849	0.0301	2.5812	0.0498
89	1.6707	0.0160	2.1204	0.0326	1.9341	0.0185	2.1491	0.0308	2.7138	0.0514
90	1.6979	0.0163	2.1917	0.0336	1.9704	0.0187	2.2169	0.0315	2.8573	0.0529
91	1.7260	0.0166	2.2677	0.0346	2.0078	0.0190	2.2883	0.0322	3.0129	0.0544
92	1.7551	0.0169	2.3484	0.0356	2.0465	0.0193	2.3637	0.0329	3.1815	0.0560
93	1.7852	0.0171	2.4345	0.0366	2.0865	0.0196	2.4432	0.0336	3.3645	0.0575
94	1.8162	0.0174	2.5260	0.0376	2.1279	0.0198	2.5272	0.0344	3.5631	0.0590
95	1.8484	0.0177	2.6235	0.0386	2.1707	0.0201	2.6158	0.0351	3.7790	0.0606
96	1.8817	0.0180	2.7275	0.0396	2.2149	0.0204	2.7094	0.0358	4.0137	0.0621
97	1.9161	0.0183	2.8382	0.0406	2.2607	0.0207	2.8082	0.0365	4.2692	0.0636
98	1.9516	0.0186	2.9562	0.0416	2.3080	0.0209	2.9127	0.0372	4.5475	0.0652
99	1.9884	0.0189	3.0821	0.0426	2.3569	0.0212	3.0231	0.0379	4.8510	0.0667
100	2.0265	0.0191	3.2165	0.0436	2.4076	0.0215	3.1398	0.0386	5.1821	0.0683

Notes:

The values for R_{35} are taken from MAGT.

The values for R_k are computed using the coefficient estimates in lines 20-24 in Table 1.

$D_k = R_k/R_{k-1} - 1.$

that is equivalent to Table 3, but there are some relevant results. Rogers et. al (1990) report a decline of 4.1 percent in 7.5 years in master athletes whose average age at the start was 62. This is a yearly fall of .0054, which compares to .0115 in Table 3 for age 64 and the events 400 meters - half marathon. Heath et. al. (1981) report between a 5 percent and 9 percent decline per decade for subjects between the ages of 50 and 62. A 5 percent decline is a yearly fall of .0049, and a 9 percent decline is a yearly fall of .0087. These numbers compare to .0096 in Table 3 for age 57. Both of these studies thus show a smaller VO_{2max} decline than the estimated decline in performance for the events 400 meters - half marathon in Table 3. Note in Table 3, however, that the derivative for the marathon is .0053 until age 60 and the derivative for 30K, 20 miles is .0045 until age 52. These numbers are close to the VO_{2max} results.

Dehn and Bruce (1972) provide an interesting regression to compare to the present results. Using a sample of ages between 40 and 69, they regress VO_{2max} adjusted for body weight on age. The coefficient estimate on age is -.362, and the estimate of the constant term is 52.741. One can compute from this regression the percentage fall in VO_{2max} at different ages, using the predicted value from this regression for the given age as the base value from which to compute the percentage. The results for selected ages compared to the results for 400 meters - half marathon in Table 3 are:

Age:	40	50	60	70	80	90	100
VO_{2max}:	.0095	.0105	.0117	.0132	.0152	.0180	.0210
Table 3:	.0080	.0080	.0104	.0132	.0160	.0187	.0215

The agreement in this case from age 60 on is remarkable, although for ages 40 and 50 the estimated decline in Table 3 is noticeably less than it is from the $\dot{V}O_{2max}$ regression. Also, estimates from the VO_{2max} regression for ages 50 and 60 are greater than the estimates from the two other studies reported above, and so the present comparisons are quite tentative. An interesting question for future work is whether the VO_{2max} results for the older ages (say 75 and above) can be used to help one estimate the slowdown rate at the older ages, where the small N_k problem is so severe.

VI. The Field Events

The small N_k problem is probably more serious for the field events than it is for the track and road racing events. This is particularly true for the shot put, discus throw, hammer throw, and javelin throw, where in many meets the weights of the relevant objects are less for older competitors. For this study only the results for the heaviest weights were used because these were the only results for which observations began at age 35.

The same procedure was followed for the field events as was followed for the other events. The log of the distance was used as the variable to be explained, and α_2 and α_5 are now expected to be negative since distance falls with age. Also, ϵ_k is expected to be mostly negative rather than mostly positive, and the frontier estimates are base on trying to force all the estimated residuals to be non positive rather than non negative. The estimation results are presented in Table 4.

The results for the high jump and triple jump are similar to each other. The are also similar to the results for the pooled sample in line 18 in Table 1, although the estimated slowdown rates are somewhat higher for the two field events. The estimated slowdown rates are considerably larger for the pole

TABLE 4

The Estimation Results for the Field Events

Event	$\hat{\alpha}_2$	SE($\hat{\alpha}_2$)	$\hat{k}_3$	SE($\hat{k}_3$)	$\hat{\alpha}_5$	SE($\hat{\alpha}_5$)	Derivative at age 50	60	75	95	SE	No. Obs.	Max Age
1 HJ	-.0093	.0009	51.5	6.2	-.00015	.00003	-.0093	-.0119	-.0163	-.0223	.017	26	90
2 PV	-.0130	.0010	64.1	2.1	-.00108	.00019	-.0130	-.0130	-.0366	-.0798	.036	31	86
3 LJ	-.0140	.0007	74.0	1.9	-.00158	.00030	-.0140	-.0140	-.0173	-.0805	.040	26	95
4 TJ	-.0125	.0012	53.1	9.6	-.00015	.00006	-.0125	-.0145	-.0189	-.0247	.022	27	83
5 SP	-.0281	.0010	-	-	-	-	-.0281	-.0281	-.0281	-	.061	23	80
6 DT	-.0280	.0013	-	-	-	-	-.0280	-.0280	-.0280	-	.070	20	78
7 HT	-.0275	.0009	-	-	-	-	-.0275	-.0275	-.0275	-	.049	21	76
8 JT	-.0273	.0010	-	-	-	-	-.0273	-.0273	-.0273	-	.059	26	77
Pooled													
9[a]	-.0278	.0005	-	-	-	-	-.0278	-.0278	-.0278	-	.060	90	80
Frontier Method													
Line[b]													
10 1	-.0095	-	62.7	-	-.00030	-	-.0095	-.0095	-.0170	-.0290	-	26	90
11 2	-.0129	-	65.8	-	-.00125	-	-.0129	-.0129	-.0358	-.0856	-	31	86
12 3	-.0135	-	75.4	-	-.00194	-	-.0135	-.0135	-.0135	-.0895	-	26	95
13 4	-.0129	-	60.5	-	-.00018	-	-.0129	-.0129	-.0180	-.0251	-	27	83
14 9	-.0266	-	-	-	-	-	-.0266	-.0266	-.0266	-	-	90	80

Notes:

[a]The pooled equations are 5-8.
[b]The frontier method used for this line above.

Max Age = oldest age used in the sample period.

HJ = high jump
PV = pole vault
LJ = long jump
TJ = triple jump
SP = shot put, 16 pounds
DT = discus throw, 2 kgs
HT = hammer throw, 16 pounds
JT = javelin throw, 800 grams

vault and the long jump, especially after the quadratic takes over at ages 64.1 and 74.0, respectively.

Sensible results using the quadratic specification could not be obtained for the other four field events -- the throwing events. The relationship between r_k and k appeared to be linear or close to linear up to about age 80, and there were not enough observations past age 80 to estimate the quadratic part with even moderate precision. There is, however, a remarkable similarity in results across the four throwing events when the linear specification is used. These results are presented in lines 5-8 in Table 4. The estimates of α_2 range only from -.0273 to -.0281. When the four events are pooled (line 9), the estimate of α_2 is -.0278. This estimated slowdown rate is larger than the rates for the other four field events expect for the pole vault and the long jump at the older ages. This estimated rate for the four throwing events seems relevant up to about age 80, but it should not be extrapolated beyond this. The data so far tell us little about what happens beyond age 80.

The frontier estimates for the first four field events and for the four throwing events pooled are presented in lines 10-14 in Table 4. As was the case for the track and road racing events, the differences between the NLS estimates and the frontier estimates are small, especially regarding the implied derivative values. The largest difference is for the high jump, where the estimate of k_3 is increased from 51.5 to 62.7 and the estimate of α_5 is changed from -.00015 to -.00030. Even here, however, the effects on the derivatives are fairly small.

The implied values of R_k for the first four field events and for the four throwing events pooled are presented in Table 5. Only values through age 80 are presented for the four throwing events pooled, for reasons discussed

TABLE 5

Estimated Age Factors for the Field Events

R_k - Projected time for age k divided by overall best time
D_k - Percentage change in R_k

	High Jump		Pole Vault		Long Jump		Triple Jump		Throwing Events	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
35	0.9381	-0.0095	0.9302	-0.0129	0.9328	-0.0135	0.9311	-0.0129	0.9381	-0.0266
36	0.9291	-0.0095	0.9182	-0.0129	0.9202	-0.0135	0.9191	-0.0129	0.9132	-0.0266
37	0.9203	-0.0095	0.9063	-0.0129	0.9078	-0.0135	0.9072	-0.0129	0.8889	-0.0266
38	0.9115	-0.0095	0.8946	-0.0129	0.8955	-0.0135	0.8955	-0.0129	0.8652	-0.0266
39	0.9028	-0.0095	0.8830	-0.0129	0.8834	-0.0135	0.8840	-0.0129	0.8423	-0.0266
40	0.8942	-0.0095	0.8716	-0.0129	0.8715	-0.0135	0.8725	-0.0129	0.8199	-0.0266
41	0.8857	-0.0095	0.8603	-0.0129	0.8597	-0.0135	0.8613	-0.0129	0.7981	-0.0266
42	0.8772	-0.0095	0.8492	-0.0129	0.8481	-0.0135	0.8502	-0.0129	0.7769	-0.0266
43	0.8689	-0.0095	0.8382	-0.0129	0.8366	-0.0135	0.8392	-0.0129	0.7562	-0.0266
44	0.8606	-0.0095	0.8274	-0.0129	0.8253	-0.0135	0.8284	-0.0129	0.7361	-0.0266
45	0.8524	-0.0095	0.8167	-0.0129	0.8142	-0.0135	0.8177	-0.0129	0.7166	-0.0266
46	0.8442	-0.0095	0.8061	-0.0129	0.8032	-0.0135	0.8071	-0.0129	0.6975	-0.0266
47	0.8362	-0.0095	0.7957	-0.0129	0.7923	-0.0135	0.7967	-0.0129	0.6790	-0.0266
48	0.8282	-0.0095	0.7854	-0.0129	0.7816	-0.0135	0.7864	-0.0129	0.6609	-0.0266
49	0.8203	-0.0095	0.7753	-0.0129	0.7710	-0.0135	0.7762	-0.0129	0.6433	-0.0266
50	0.8125	-0.0095	0.7652	-0.0129	0.7606	-0.0135	0.7662	-0.0129	0.6262	-0.0266
51	0.8047	-0.0095	0.7553	-0.0129	0.7503	-0.0135	0.7563	-0.0129	0.6096	-0.0266
52	0.7971	-0.0095	0.7456	-0.0129	0.7402	-0.0135	0.7466	-0.0129	0.5934	-0.0266
53	0.7894	-0.0095	0.7359	-0.0129	0.7302	-0.0135	0.7369	-0.0129	0.5776	-0.0266
54	0.7819	-0.0095	0.7264	-0.0129	0.7203	-0.0135	0.7274	-0.0129	0.5623	-0.0266
55	0.7745	-0.0095	0.7170	-0.0129	0.7106	-0.0135	0.7180	-0.0129	0.5473	-0.0266
56	0.7671	-0.0095	0.7077	-0.0129	0.7010	-0.0135	0.7088	-0.0129	0.5328	-0.0266
57	0.7598	-0.0095	0.6986	-0.0129	0.6915	-0.0135	0.6996	-0.0129	0.5186	-0.0266
58	0.7525	-0.0095	0.6896	-0.0129	0.6822	-0.0135	0.6906	-0.0129	0.5048	-0.0266
59	0.7453	-0.0095	0.6806	-0.0129	0.6729	-0.0135	0.6817	-0.0129	0.4914	-0.0266
60	0.7382	-0.0095	0.6718	-0.0129	0.6639	-0.0135	0.6729	-0.0129	0.4783	-0.0266
61	0.7312	-0.0095	0.6632	-0.0129	0.6549	-0.0135	0.6641	-0.0131	0.4656	-0.0266
62	0.7242	-0.0095	0.6546	-0.0129	0.6460	-0.0135	0.6552	-0.0134	0.4533	-0.0266
63	0.7172	-0.0097	0.6461	-0.0129	0.6373	-0.0135	0.6461	-0.0138	0.4412	-0.0266
64	0.7098	-0.0103	0.6378	-0.0129	0.6287	-0.0135	0.6370	-0.0141	0.4295	-0.0266
65	0.7020	-0.0109	0.6295	-0.0129	0.6202	-0.0135	0.6277	-0.0145	0.4181	-0.0266
66	0.6940	-0.0115	0.6210	-0.0134	0.6118	-0.0135	0.6184	-0.0149	0.4070	-0.0266
67	0.6855	-0.0121	0.6112	-0.0159	0.6035	-0.0135	0.6090	-0.0152	0.3961	-0.0266
68	0.6768	-0.0127	0.5999	-0.0184	0.5954	-0.0135	0.5995	-0.0156	0.3856	-0.0266
69	0.6678	-0.0133	0.5874	-0.0209	0.5873	-0.0135	0.5900	-0.0159	0.3754	-0.0266
70	0.6585	-0.0139	0.5736	-0.0234	0.5794	-0.0135	0.5804	-0.0163	0.3654	-0.0266
71	0.6489	-0.0145	0.5588	-0.0259	0.5716	-0.0135	0.5707	-0.0166	0.3557	-0.0266
72	0.6391	-0.0151	0.5430	-0.0284	0.5638	-0.0135	0.5611	-0.0170	0.3462	-0.0266
73	0.6290	-0.0158	0.5262	-0.0309	0.5562	-0.0135	0.5513	-0.0173	0.3370	-0.0266
74	0.6188	-0.0163	0.5087	-0.0333	0.5487	-0.0135	0.5416	-0.0177	0.3281	-0.0266
75	0.6083	-0.0170	0.4904	-0.0358	0.5413	-0.0135	0.5318	-0.0180	0.3193	-0.0266
76	0.5976	-0.0176	0.4716	-0.0383	0.5327	-0.0159	0.5220	-0.0184	0.3109	-0.0266

TABLE 5 (continued)

	High Jump		Pole Vault		Long Jump		Triple Jump		Throwing Events	
	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k	R_k	D_k
77	0.5867	-0.0182	0.4524	-0.0408	0.5221	-0.0198	0.5122	-0.0187	0.3026	-0.0266
78	0.5757	-0.0188	0.4328	-0.0433	0.5098	-0.0237	0.5025	-0.0191	0.2945	-0.0266
79	0.5646	-0.0194	0.4130	-0.0458	0.4957	-0.0275	0.4927	-0.0194	0.2867	-0.0266
80	0.5533	-0.0200	0.3930	-0.0483	0.4802	-0.0314	0.4829	-0.0198	0.2791	-0.0266
81	0.5419	-0.0206	0.3731	-0.0508	0.4632	-0.0353	0.4732	-0.0202	-	-
82	0.5304	-0.0212	0.3532	-0.0533	0.4451	-0.0392	0.4635	-0.0205	-	-
83	0.5189	-0.0218	0.3335	-0.0558	0.4259	-0.0430	0.4538	-0.0209	-	-
84	0.5072	-0.0224	0.3141	-0.0582	0.4059	-0.0469	0.4442	-0.0212	-	-
85	0.4956	-0.0230	0.2950	-0.0607	0.3853	-0.0508	0.4346	-0.0216	-	-
86	0.4839	-0.0236	0.2764	-0.0632	0.3643	-0.0547	0.4251	-0.0219	-	-
87	0.4722	-0.0242	0.2582	-0.0657	0.3429	-0.0585	0.4156	-0.0223	-	-
88	0.4605	-0.0248	0.2406	-0.0682	0.3215	-0.0624	0.4062	-0.0226	-	-
89	0.4487	-0.0254	0.2236	-0.0707	0.3002	-0.0663	0.3968	-0.0230	-	-
90	0.4371	-0.0260	0.2072	-0.0732	0.2792	-0.0702	0.3876	-0.0233	-	-
91	0.4254	-0.0266	0.1915	-0.0757	0.2585	-0.0740	0.3784	-0.0237	-	-
92	0.4138	-0.0272	0.1766	-0.0781	0.2383	-0.0779	0.3693	-0.0240	-	-
93	0.4023	-0.0278	0.1623	-0.0806	0.2188	-0.0818	0.3603	-0.0244	-	-
94	0.3909	-0.0284	0.1488	-0.0831	0.2001	-0.0857	0.3513	-0.0248	-	-
95	0.3795	-0.0290	0.1361	-0.0856	0.1822	-0.0895	0.3425	-0.0251	-	-
96	0.3683	-0.0296	0.1241	-0.0881	0.1652	-0.0934	0.3338	-0.0255	-	-
97	0.3572	-0.0302	0.1129	-0.0906	0.1491	-0.0973	0.3252	-0.0258	-	-
98	0.3461	-0.0308	0.1024	-0.0931	0.1340	-0.1012	0.3167	-0.0262	-	-
99	0.3352	-0.0315	0.0926	-0.0956	0.1199	-0.1050	0.3083	-0.0265	-	-
100	0.3245	-0.0321	0.0835	-0.0981	0.1069	-0.1089	0.3000	-0.0269	-	-

Notes:

The values for R_{35} are taken from MAGT.

The values for R_k are computed using the coefficient estimates in lines 1-9 in Table 1.

$D_k = R_k/R_{k-1} - 1$.

above. The estimates in lines 10-14 in Table 4 were used for these values, which are the estimates based on the frontier procedure. The values for R_{35} for each event were taken from MAGT.

Comparing Tables 3 and 5, almost all the derivatives are larger in absolute value in Table 5. Men seem to slow down faster in the field events than they do in the track and road racing events. The two exceptions to this are 1) the high jump and triple jump at the older ages, where the slowdown rates are not out of line with the rates for the pooled events in Table 3, and 2) the marathon, where the slowdown rates at the older ages are high relative to the rates for the high jump, triple jump, and the throwing events. These two exceptions pertain only to ages beyond about 80, however, and it seems clear that for ages below 80 the slowdown rate is greater for the field events than it is for the running events.

V. Conclusion

Since I am an economist, one might ask if the above has anything to do with economics? From an economist's perspective, is this study simply an exercise in applying some econometric techniques to a data set? Maybe. But I am struck from looking at the numbers in Table 3 how small the slowdown rates are. For example, using the values of R_k for the events 400 meters - half marathon, a man of 85 is only 49 percent slower than he was at age 55 (1.8007 versus 1.2102). (Presumably the numbers are similar for women.) Do these numbers say something about productivity loss with age, about the optimal wage profile with age, about retirement policies? Maybe most societies are too pessimistic about the losses from aging and have passed various laws dealing with old people under incorrect assumptions? Perhaps old people can be used in more rigorous ways than has heretofore been the case? Or maybe the numbers in Table 3 are only of interest to old runners as they run ever more slowly into the sunset.

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MEMO

To: Old runners, people interested in old runners, race officials

From: Ray C. Fair

Subject: How Fast Do Old Men Slow Down

Date: May 28, 1991

The enclosed paper presents estimates of the rate at which men slow down with age in track, field, and road racing events. The paper is fairly technical, but Tables 2, 3, and 5 can be understood without reading the technical material. Table 2 shows that the age-factors in Masters Age-Graded Tables (MAGT) are excessively variable and biased against older runners. Tables 3 and 5 present the age-factors implied by this study.

Table 3 allows an old runner to estimate his projected times by age. To use the table, first pick the best time you think you could ever have run in the event, when you were in your 20's or early 30's. This could be either your actual time if you were running in these years or your best guess as to what this time would have been had you been running. Then multiply this time by the number in Table 3 that corresponds to your current age. This gives your projected time at your current age.

Race officials should consider using the age-factors in Table 3 in place of the age-factors in MAGT for age-graded races. The age-factors in Table 3 are more closely tied to actual best performances by age than are the MAGT age-factors, and they are not excessively variable.

Two examples of another way in which Table 3 can be used may be of interest. First, Mike Tymn in the February 1991 issue of National Masters News wrote about his 10K times. At the age of 41 he ran 31:38. He then had some bad years, but came back strong at the age of 53 to run 34:40. Given that he ran 31:38 at age 41, what would Table 3 predict he should run at age 53? The age-factor in Table 3 is 1.0791 for age 41 and 1.1889 for age 53, which is a 10.175 percent increase between the two ages. Tymn's age-53 time is thus predicted to be 10.175 greater than his age-41 time. This would be a time of 34:51, which is quite close to his actual time at age 53 of 34:40. Table 3 thus predicts very well in this case. To see how Tymn might do ten years hence, the age-factor for age 63 in Table 3 is 1.3135, which is a 10.48 percent increase from age 53. Given his age-53 time of 34:40, this would be a time at age 63 of 38:18.

The other example concerns Frank Shorter. Shorter has run the 4.55-mile Bemis-Forslund Pie Race many times, the first when he was 15. His best time occurred at age 32, when he ran 20:54. Last year at age 43 he ran 22:40. (See Runner's World, May 1991, p. 118, for these numbers.) What would Table 3 predict he should have run at age 43? The age-factor in Table 3 for age 43 is 1.0965. If we assume that Shorter's age-32 time of 20:54 is the best he could ever have run, his predicted time for age 43 is 1.0965 times 20:54, which is a time of 22:55. This is quite close to his actual time at age 43 of 22:40, and so again Table 3 predicts well. The age-factor for age 80 in Table 3 is 1.6568, which means that Shorter should be able to run the Pie Race when he is

80 in a time of 34:38 (1.6568 times 20:54) assuming that he is not sick or injured and that he stays in top shape for his age.

Table 5 presents the age-factors for the field events. These again may be of interest to race officials organizing age-graded meets.

Finally, a word of caution. The age-factors in Tables 3 and 5 are less reliable after about age 80 than they are before. The age-factors after age 80 will probably change the most in the future when the equations used in this study are reestimated in light of new age records that are set. Most likely the age-factors after age 80 will come down, but it is unclear at this time whether they will come down a lot or only a little.

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